# A New Adaptive Sliding Mode Control Law of Chaotic Systems to Synchronization

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**Abstract:** In this research, a new adaptive sliding mode control scheme which combines two algorithms mentioned before is introduced to synchronize chaotic Lorenz models and the synchronization of chaos by designing a new method is presented. As the simulations show, the proposed controller is efficient to synchronize chaotic systems and has better performance compared to former methods. These results are obtained from Integral of Squared Sliding Surface Signal and Integral of Squared Sliding Control Signal criteria. Furthermore, the boundary layer is proposed in the new control algorithm to avoid the chattering phenomenon on behaviors.

**Keywords:** Chaos Syncronization, Lorenz Model, Adaptive, Sliding Mode Control

## 1. Introduction

Chaos phenomenon is very interesting area in the nonlinear systems described by differential equations which can be extremely sensitive to initial conditions. The concept of controlling and synchronization of chaotic systems have been attracted by researchers since the early 1990s. In recent years, several techniques of chaos control and synchronization have been applied such as linear and nonlinear feedback control [1,2], adaptive control [ 3-5 ] and sliding mode control [ 6-8 ].

In the past decades, the sliding mode control (SMC) has been effectively applied to control systems with uncertainties because of the nature of robustness of sliding mode [9]. The adaptive techniques have been also applied to control and synchronize chaotic systems [10,11]. Recently, researchers have utilized the adaptive control with sliding mode technique for many engineering systems to decrease the chattering in pure SMC and smooth the output from a sliding mode controller. Dadras and Momeni [ 12 ] proposed an adaptive sliding mode control to synchronize master-slave chatotic systems. This scheme reduces the chattering phenomenon and guarantees stability in presence of parameter uncertainties and external disturbance. Roopaei et al. [ 13 ] developed an adaptive sliding mode controller to stabilize the novel class chaotic system.

In this research, a new adaptive sliding mode control scheme which combines two algorithms mentioned before is introduced to synchronize chaotic Lorenz models and the synchronization of chaos by designing a new method is presented.

## 2. System Description and Problem Formulation

Considering system uncertainties a class of the following two *n*-dimensional chaotic systems can be written as:

$$\dot{x}_i = x_{i+1} \qquad 1 \le i \le n-1 \qquad (1)$$
$$\dot{x}_n = f(x,t) \qquad x = [x_1, x_2, \dots, x_n] \in R^n$$

$$\dot{y}_i = y_{i+1} \qquad 1 \le i \le n-1 \qquad (2)$$
$$\dot{y}_n = f(y,t) + \Delta f(y) + u \qquad y = [y_1, y_2, \dots, y_n] \in R^n$$

Where $u \in R$ is the control input, $f$ is a given nonlinear function of $x, y, t$. $\Delta f(y)$ is an uncertain term representing the unmodeled dynamics or structural variation of (2).

In this paper, the chaos synchronization is considered as a model tracking problem in which the slave system (2) can track the master system (1) asymtotically. The two coupled system to be synchronized by designing an appropriate control *u(t)* in system (2) such that

$$\lim_{t\to\infty} \|y(t) - x(t)\| \to 0 \tag{3}$$

where $\|.\|$ is the Euclidian norm of a vector. Let us define the error states between (1) and (2) such as:

$$e_1 = y_1 - x_1, \dots., e_n = y_n - x_n \tag{4}$$

The problem is to realize the synchronization between two chaotic systems is to choose a control law *u(t)* to make error states converge to zero. Here an adaptive sliding mode control design is used to achieve this objective.

## 3. Design of Adaptive Sliding Mode Control via Synchroziation Problem

To propose a new adaptive control algorithm for synchronization of two chaotic systems are described in (1) and (2), an adaptive switching surface is considered as

$$s(t) = e_n(t) + \varphi(t) \tag{5}$$

Because this surface was considered as in [13] referred study as shown below

$$s(t) = x(t) + \varphi(t) \tag{5a}$$

(5a) was designed for a control problem and we changed it to equation (5) in order to adapt to our synchronization problem and so we obtained (5).

In (5) $e_n(t)$ is the $n$ – dimensional system error state and $\varphi(t)$ is an adaptive function given by

$$\dot{\varphi} = e_i(x, y)e_{i+1}(x, y) + \alpha e_i + \rho e_{i+1} \tag{6}$$

where $\alpha$ and $\rho$ are assumed to be the arbitrary constants. This adaptive function is also from Poopaei study [ ] and here it is adapted to the synchronization problem. When the system operates in sliding mode, it satisfies the following condition:

$$s(t) = e_n(t) + \varphi(t) = 0 \tag{7}$$

Differentiating Eq.(7), leads to the following:

$$\dot{s}(t) = \dot{e}_n(t) + \dot{\varphi}(t) = 0 \tag{8}$$

The equivalent control law can be obtained by utilizing (4) and (8) as shown below:

$$u_{eq} = -f(y, t) - \Delta f(y) + f(x, t) - \dot{\varphi} \tag{9}$$

(The open version of (10) is given in Section 5 as equation (30))

The next step is to design the reaching mode control scheme which drives the system trajectories on to the sliding surface ($s=0$). In the proposed method, we aim to derive a new algorithm combining Dadras and Roopaei methods to increase the control performance. Therefore the overall control signal has the form of following:

$$u(t) = u_{eq} - \gamma\varepsilon(\lambda, s) + k_s sign(s) \tag{10}$$

where $\varepsilon(\lambda, s)$ is a hyperbolic function (Dadras):

$$\varepsilon(\lambda, s) = \tanh(\lambda s) \tag{11}$$

and the Dadras adaptive law is:

$$\dot{\gamma} = m_1 e\varepsilon(\frac{\delta x}{\delta u}) \tag{12}$$

$$\dot{\lambda} = m_2 e(1 - \varepsilon^2)s(\frac{\delta x}{\delta u}) \tag{13}$$

The Roopaei adaptive law in combined overall control signal (10) is:

$$\dot{k}_s = -\mu|s| \tag{14}$$

where $\mu$ is a positive constant number.

## 4. Case Study and Simulation Results

This section of the paper presents the case study to verify and demonstrate the effectiveness of the proposed control scheme. The simulation results are carried out using the MATLAB software. The fourth order Runge-Kutta integration algorithm was performed to solve the differential equations.

Consider the chaotic Lorenz master-slave systems as follows:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = cx_1 - x_1x_3 - x_2 \\ \dot{x}_3 = x_1x_2 - bx_3 \end{cases} \tag{15}$$

and

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) \\ \dot{y}_2 = cy_1 - y_1y_3 - y_2 + \Delta f(Y) + u(t) \\ \dot{y}_3 = y_1y_2 - by_3 \end{cases} \tag{16}$$

where a,b, c are the positive contants, $\Delta f(y)$ is the uncertainty term and *u(t)* is the adaptive sliding mode control law.

The error states defines as given below

$$e1 = y1 - x1,\ e2 = y2 - x2,\ e3 = y3 - x3 \tag{17}$$

The sliding surface and adaptive funtion are choosen respectively, regarding Eq. (5) and (6):

$$s = e_1 + e_2 + \varphi \tag{18}$$

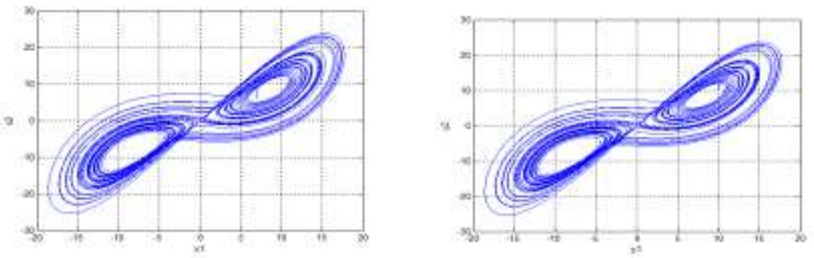
$$\dot{\varphi} = e_1e_2 + \alpha e_1 + \rho e_2 \tag{19}$$

where $\alpha$ and $\rho$ are the positive constants. The continious part of control law ( $u_{eq}$) is obtained from $\dot{s} = 0$ as shown below:

$$u_{eq} = a(x_2 - x_1) - a(y_2 - y_1) - cy_1 + y_1 y_3 + y_2 + cx_1 - x_1 x_3 - x_2 - (e_1 e_3 + ae_1 - e_2 + \rho e_2 ) \quad (20)$$

The simulation is done with the initial value $[x_1\ x_2\ x_3]^T = [7\ 6\ 12]^T$ , $[y_1\ y_2\ y_3]^T = [10\ 10\ 10]$ and system parameters are $a$=10, $b=\frac{8}{3}$ , $c$=28. Control parameters for Dadras adaptive law are choosen as $m_1 = m_2 = 5$. In addition the second adaptive law is used to update the $k_s$, $\dot{k}_s = -0.1|s|$ . The slave system is perturbed by an uncertainty term:

$$\Delta f = 0.5 - \sin(\pi y_1) \sin(2\pi y_2) \sin(3\pi y_3)$$



(a) (b)

Fig. 1. Phase Plane of two Lorenz model after synchronization (a) master (b) slave

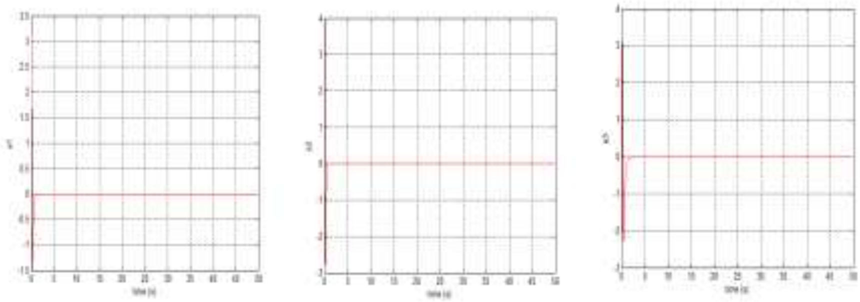


Fig. 2. The Error States

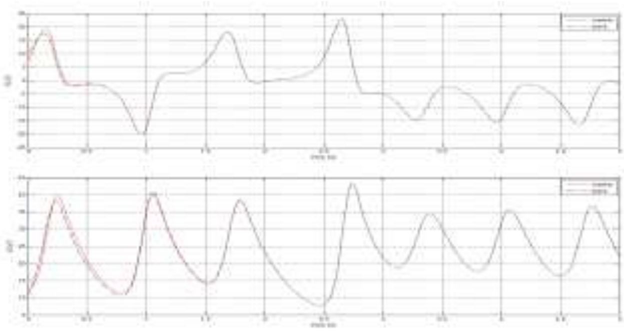


Fig. 3. Synchronization of state variables with proposed control law

**Table 1**

| | Proposed Control | Proposed Control(with boundary layer) | Dadras Control | Dadras Control(with boundary layer) |
|---|---|---|---|---|
| $\int_0^5 s^2(t)dt$ | $0.99.10^{-6}$ | $1.05.10^{-5}$ | $1.5.10^{-5}$ | $1.13.10^{-5}$ |
| $\int_0^5 u^2(t)dt$ | 1.15 | 0.2013 | 0.2544 | 0.2588 |

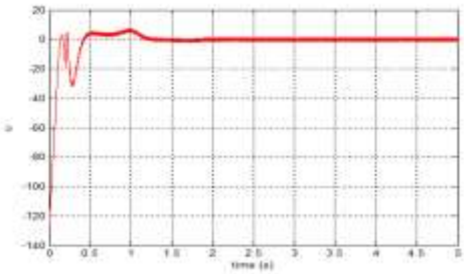



Fig.4. Control input for proposed algorithm without boundary layer

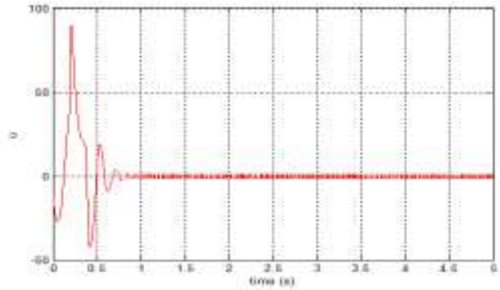



Fig.5.  Control input for Dadras algorithm without boundary layer

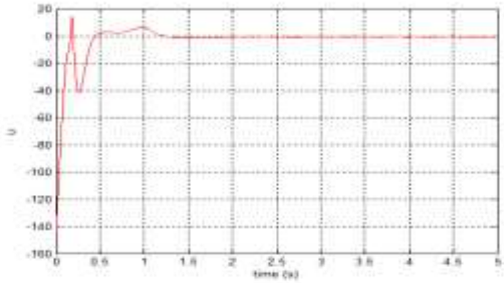



Fig.6.  Control input fo rproposed algorithm with boundary layer

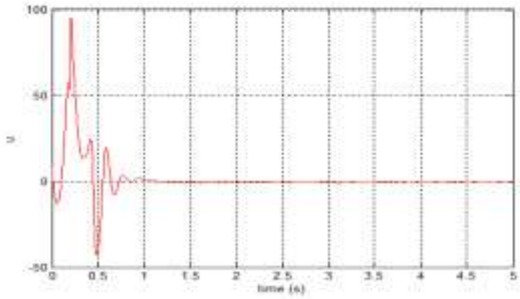



Fig.7.  Control input for Dadras algorithm with boundary layer

The first parts of the simulation results are shown in Figs. 1-3 under the proposed adaptive sliding mode control (ASMC). Fig 1. shows the phase plane of master-slave Lorenz systems whereas Fig 2. exhibits the error states and Fig 3. presents the state variables after synchronization after ASMC application. From these results, it can be concluded that the obtained theoretic results are feasible and efficient for synchronizing Lorenz chaotic systems under uncertainty.

In the second part of simulation study, proposed adaptive sliding mode control is compared with Dadras method. "*Integral of Squared Sliding Surface Signal*" and " *Integral of Squared Sliding Control Signal*" criteria are used. Table 1 presents the results. Referring to Table 1 and Figs 4-7, it can be concluded that in the case of using a boundary layer, our combined adaptive sliding mode control scheme can manage better performance compared to Dadras method.

## 6. Conclusion

This work presents the synchronization of chaos by designing a new adaptive sliding mode controller. As the simulations show, the proposed controller is efficient to synchronize chaotic systems and has better performance compared to Dadras method. These results are obtained from *Integral of Squared Sliding Surface Signal*" and " *Integral of Squared Sliding Control Signal*" criteria. Furthermore, the boundary layer is proposed in the new control algorithm to avoid the chattering phenomenon on behaviors.

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