

Book of Abstracts

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Plenary Sessions

Professor Alexander Alekseevich Potapov (on the occasion of the 70th birthday)

Potapov Alexander Alekseevich (was born in 1951.05.04) finished Ryazan State Radio Engineering Institute (Department of Radio Engineering) in 1974 and M.V. Lomonosov Moscow State University (Department of Physics) in 1979, Dr. Sc. (Phys.-Math.) - (1994), Chief Research of V.A. Kotelnikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences (RAS).

Academician of Russian Academy of Natural Sciences (2007, Russia); Academician of the Academy of engineering science of A.M. Prokhorov (2008, Russia), President of the Sino-Russian Laboratory of Informational Technologies and Signals Fractal Processing (2011), Guangzhou, China. Potapov A.A. - Honorary Professor of the Eurasian National University (Kazakhstan, Astana, 2010), Honorary Professor of the University of Jinan (China, Guangzhou, 2011). Honorable radio operator of the Russian Federation (2006). He was awarded 18 medals or outstanding achievements in the field of engineering sciences and designing of fundamentally new fractal information technologies.

Author of the basic researches in the field of fractal and textural approaches to radio physics problems, radio engineering, radar and broad spectrum of adjacent scientific and technology path. He has been working at this field from 1979. Initiator of the pioneer researches and developments of fractal theory application in radio physics, scaling effects and fractal operators in radio systems in Russia. The concept of creating fundamentally new fractal radio systems, fractal sensors and fractal element base is proposed. He was (1997 and 2000) awarded the Government scientific grant twice. Now, based on the pioneering work of A.A. Potapov with his students at the V.A. Kotelnikov IREE RAS, a new fundamental direction "Fractal radio physics and fractal radio electronics: designing fractal radio systems" was formed and a unique Russian scientific school of fractal methods, well known in the world, was created.

Several international projects have been also conducted (the USA and China).

A working model of the world's first fractal nonparametric radar signal detector was created in V.A. Kotelnikov IREE RAS in 2005. A rigorous electrodynamic calculation of numerous types of fractal antennas was carried out, the design principles of which underlie fractal frequency-selective surfaces and volumes (fractal "sandwiches"). For the first time, a model of a "fractal" capacitor as a fractal impedance, as well as fractal labyrinths for the synthesis of microwave structures, was proposed and implemented. In 1997, for the first time, methods of fractal modulation and fractal broadband and ultrawideband signals were developed, including the H - signals developed by the author. In 2015 A.A. Potapov, for the first time in world practice, discovered, proposed, substantiated and developed the principles of a new type and new method of radiolocation, namely, fractal-scaling or scale-invariant radar. The efficiency of functionals, which are determined by the topology, fractional dimension and texture of the received multidimensional signal, for the synthesis of fundamentally new non-energy detectors of low-contrast objects against the background of noise, has been proved. The postulates of fractal radiolocation have been developed. The ways of transition from the created type of fractal radar to cognitive radar and to quantum radar are investigated.

The results of A.A. Potapov's scientific efforts on the fractal and textural processing of information in the presence of high-intensity noise, and also on fractal radio systems and fractal radio elements are published in summary reports of Presidium of Russian academy of science (RAS) (Scientific achievements of RAS. - M: Nauka, 2008, 2010, 2012 and 2013) and in the Report to the Government of the Russian Federation (2012). All these world priority results allow moving on the new level of information structure of the real non-Markov signals and fields.

A.A. Potapov published personally and in co-authorship over 1150 scientific works in domestic and international publications, including over 45 monographs and chapters in books in Russian and English, 2 patents and 9 textbooks. At international (23 countries) and all-Russian conferences he made more than 500 reports, of which more than 300 plenary and sectional reports as an invited speaker.

He is the founder and editor-in-chief of the magazine "Nelineinyi mir (Non-linear world, Moscow)" (2003), member of editorial boards of 13 Russian and international magazines.

The biography of Potapov Alexander Alekseyevich is published in the encyclopedia "Who is who in Russia" (Verlag fur Personnenzyklopadien AG - Switzerland) on personal request of publishing house (2009–2010). A.A. Potapov is the author of the first in Russia monograph "Fractals in radio physics and radar" (M: Logos, 2002, 664 pp., number of copies 1000), which was edited and added in 2005 (A.A. Potapov. Fractals in radio physics and radar: Sampling topology. – M: University book, 2005, 848 pp., number of copies 2000). These two monographs became the reference books for the scientists from many fields. A.A. Potapov is also the author and co-author of series of monographs about radar and fractal application in science and engineering. He developed lectures on fractal and wavelet application in radar which he reads in the Centre of specialists training in Concern REI Systems (Academic A.L. Mints REI and public corporation NPK NIIDAR). A.A. Potapov is the member of organizing committees of many international and Russia conferences.

Based on the monographs of A.A. Potapov lecture courses "Fractals in Statistical Radio physics", "Statistical Theory of Fractal Radiolocation", "Fractals in Radiophysics and Radiolocation", "Statistical Fractal Radio Engineering", "Fractals in Mechanical Engineering", etc. delivered at various universities in Russia, neighboring countries and China.

Lecturing on the technologies which he developed in V.A. Kotelnikov IREE RAS and reports on project MNTC in USA (Washington, New-York, Huntsvill, Atlanta, Franklin) in 2000 and 2005, in China (2011) and on many international conferences (England, USA, Canada, Netherlands, Austria, Germany, France, Spain, Italy, Hungary, Greece, Turkey, Scotland, Switzerland, Sweden, Mexico, China, Serbia, Montenegro, Bulgaria, Kazakhstan, Belarus, Ukraine) brought him good repute in circle of international scientific community. In December 2005 American specialists noticed in the official letter addressed to academic and IREE RAS director Y.V. Gulyaev that "... Seminars were very interesting and confirmed high scientific qualification of doctor A. Potapov. Radar technologies which were presented by doctor Potapov are based on the fractal theory and are the new ones. The significance of these investigations for international community of specialists and scientists is evident". In 2005, a significant scientific meeting of A.A. Potapov with the founder of fractal geometry B.B. Mandelbrot occurred, who warmly supported the work of Professor A.A. Potapov.

High scientific erudition, efficiency, adherence to principles, purposefulness, a great sense of responsibility for the work he is engaged

in, and an all-consuming love for science brought A.A. Potapov well-deserved authority and wide popularity among scientists.

Scientific Interest: statistical radio physics, dispersion and diffraction of electromagnetic waves, radio location, image and signal processing and recognition, deterministic chaos, modern topology, fractal analysis, fractal operators, scaling effects, fractal antennas, fractal electrodynamics, photonics, metamaterials and metasurfaces.

Forecasting tools for chemical damage and consolidation of building materials: modelling and simulations

G. Bretti, M. Ceseri, R. Natalini

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This is an interdisciplinary work on chemical aggression of building materials (stones, concrete) due to the presence of water and pollutant agents such as the carbon dioxide. In the last few years we studied some mathematical models to assess the evolution of damage due to external agents on stones used in buildings and historical monuments. Besides, these models can be modified to take into account also the effect of some consolidation procedures, using different chemical compounds. In this talk I will present a model for the chemical aggression of concrete and stones by CO₂, which are based on a physico-chemical description in terms of partial differential equations, and their agreement with experimental data.

Translating ecology into economics

Harold M. Hastings, Tai Young-Taft

Bard College at Simon's Rock, 84 Alford Road, Great Barrington MA., USA

A seminal paper by Robert M. May, Simon A. Levin & George Sugihara ("Ecology for bankers," *Nature*, 2008) showed how the study of "ecosystems [can] inform the design of financial networks in, for instance, their robustness against perturbation." Our presentation will build upon these ideas to translate an analysis of stability concepts and scaling rules from ecology to economics: for example, the relationship between

MacArthur's (1955) and Hutchinson's (1959) argument that stability increases with increasing complexity, in contrast to May's argument that stability decreases with increasing complexity, and more generally resource partitioning in ecosystems, using the ansatz Economics = ecology + currency.

Keywords: Ecology, economics, network science, stability, complexity, scaling.

Nonlinear Phenomena in the Dynamics of a Class of Rolling Pendulums: A Trigger of Coupled Singularities

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In the introductory part of the lecture, an overview of nonlinear differential equations of heavy ball and heavy thin disk rolling along curvilinear paths and surfaces of different shapes will be presented. An overview of nonlinear equations of phase trajectories will be given for a number of special rolling points on spherical surfaces, on a cone and on a torus. For a number of nonlinear dynamics of ball and thin disk rolling along curvilinear paths, phase portraits will be presented with the definition of the term generalized rolling pendulum. The subject of the lecture is the identification and presentation of nonlinear phenomena in the nonlinear dynamics of a class of generalized rolling pendulums, whose heavy bodies roll along curvilinear paths, lying in a vertical plane, rotating around a vertical axis, at a constant angular velocity. The bifurcation parameter of coupled rotations is identified. The bifurcation of the position of stable equilibrium of the generalized rolling pendulum and the corresponding representative singular points of the type of the stable center is described, as well as the stratification and transformation of phase trajectories in the phase portrait of nonlinear dynamics of the generalized rolling pendulum in the Earth's gravitational field, and along curvilinear route in rotate vertical plane around vertical axis at a constant angular velocity. Two theorems of trigger of coupled singularities and a homoclinic orbit in the form of the number "eight" are presented. A series of phase portraits for different coefficients of curvilinear paths described by polynomials of the eighth degree is presented and sets of transformed phase trajectories and

homoclinic orbits in the form of the number "eight" are presented, which include one or more triggers of coupled singular points.

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Keywords: Generalized rolling pendulum, Bifurcation, Trigger of coupled singularities, Curvilinear rolling path, Phase trajectory portrait, Homoclinic orbit, Theorem.

The wave gravitational field arising and wave processes under orbital motion of a gravitating body

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This work investigates wave processes in orbital motion of a body under its gravitational interaction with a central body based on the statistical theory as well as the theory of retarded gravitational potentials. The statistical theory for a cosmogonical body forming (so-called spheroidal body) has been proposed in our previous works. Starting the conception

for forming a spheroidal body inside a gas-dust protoplanetary nebula, this theory solves the problem of gravitational condensation of a gas-dust protoplanetary cloud from the point of view of planetary formation in its own gravitational field. The proposed statistical theory predicts statistical oscillations of orbital motion of planets around stars in forming protoplanetary system. Indeed, as known, the Alfvén–Arrhenius’s radial and axial oscillations modify the forms of planetary orbits. Here we explain how the stability of the orbital body (a planet) moving around the central gravitating body (a star) is reached by the wave gravitational interaction between them. Using the statistical theory of cosmogonical body formation we find that periodic temporal deviation of the gravitational compression function of a spherically symmetric spheroidal body (under the condition of mechanical quasi-equilibrium) induces an additional periodic force of Alfvén–Arrhenius. Within the framework of the developed theory of retarded gravitational potentials, the formula of the wave gravitational potential is also confirmed. As shown, a wave exchange of gravitational energy occurs between the central body and the orbitally moving body. In this connection, its orbital motion should be considered in a fast oscillating field, i.e. such motion of this body simultaneously under the action of constant field and time-varying field with a high frequency.

Keywords: Gas-dust protoplanetary nebula, Spheroidal bodies, Gravitational condensation, Exoplanetary systems, Stability of planetary orbits, Alfvén–Arrhenius’ oscillating forces, Wave gravitational field, Wave exchange of gravitational energy.

Isoscattering Chains of Graphs and Networks

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We discuss the latest breakthrough achievements in the field of isospectral and isoscattering systems. We show that it is possible to generalize a famous question of Mark Kac "Can one hear the shape of a drum?", originally posed in the case of isospectral dissipationless systems, to the case of infinite strings of open graphs and networks. We begin with identifying isoscattering chains of quantum graphs possessing n units and $2n$ infinite external leads. Then, we discuss the principles of

designing large graphs and networks for which the isoscattering properties are preserved for $n \rightarrow \infty$. In order to confirm experimentally the theoretical predictions we design and investigate $n=2$ units, four-leads microwave networks. In the mathematical approach our work goes beyond prior results [1-3] by demonstrating that using a trace function one can address the unsettled until now problem of whether scattering properties of open complex graphs and networks with many external leads are uniquely connected to their shapes. The application of the trace function reduces the number of required entries to the scattering matrices S of the systems to the diagonal elements. It paves the way towards the future experimental analysis even more complex isoscattering systems and possible applications.

Acknowledgements: This work was supported in part by the National Science Centre, Poland, Grant No. 2016/23/B/ST2/03979.

References:

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Keywords: Quantum graphs, Microwave networks, Isospectrality, Isoscattering, Open systems, Simulations.

Chaos in cable and beams

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This work, based on [1,2], is a review of occurrence of chaos in mechanical systems, in particular in cable and beams which have the common characteristic to be one dimensional in space and to be pervasive in civil, mechanical and other branches of engineering. Attention is paid to

different mechanical models, from the one side, and to different dynamical outcomes, from the other side, with special attention to chaotic dynamics. The underlying goal is to show that chaos is not only a fascinating research topic, but it also has a strong relevance in engineering applications, where sometimes it can also be exploited for specific practical needs (e.g. chaos control for increasing robustness).

References:

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On the Origin of the Universe: Chaos or Cosmos?

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I would like to consider the Universe according to the standard Big Bang model, including various quantum models of its origin. In addition, using the theory of nonlinear dynamics, deterministic chaos, fractals, and multifractals I have proposed a new hypothesis, Ref. [1]. Namely, I have argued that a simple but possibly nonlinear law is important for the creation of the Cosmos at the extremely small Planck scale at which space and time originated. It is shown that by looking for order and harmony in the complex real world surrounding us these modern studies give new insight into the most important philosophical issues beyond classical ontological principles, e.g., by providing a deeper understanding of the age-old philosophical dilemma (Leibniz, 1714): why does something exist instead of nothing? We also argue that this exciting question is a philosophical basis of matters that influence the meaning of human life in the vast Universe.

References:

Macek, W. M., *The Origin of the World: Cosmos or Chaos?*, UKSW Scientific Editions, 2020, ISBN 978-83-8090-686-0, e-ISBN 978-83-8090-687-7.

Keywords: Chaos, Cosmos, Universe, Creation.

Shedding light on complex locomotion

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C. elegans is a widely studied, fully developed model organism with a digestive system and the ability to learn, and a fully mapped neural network consisting of 302 neurons (<http://www.wormbook.org/>). In this work, we characterize its locomotion as low dimensional chaos using dynamic diffraction methods developed in the Vassar Applied Optics Lab. By placing the worms in a cuvette, dynamic diffraction converts the locomotion in 3D into a time series obtained from optical fluctuations in the dynamic diffraction pattern. We calculate the largest Lyapunov exponent (LLE) of this time-series. We use this calculation to characterize the locomotory predictability of the microscopic species. The computational methods used to perform these techniques as well as the experimental setup will be presented. Chaotic markers are evident in the largely deterministic and low-noise dynamics with an LLE of 1.29 ± 0.02 1/s for the wildtype as well as the recurrence plots. This low dimensional chaotic behavior indicates predictability over about three times the Lyapunov time - in this case roughly ~ 2.3 s. We compare these results with harmonic sinusoidal models and locomotion that is generated by a network of FitzHugh-Nagumo equations. The underlying cause of the differences in LLE between different *C. elegans* is still unknown. Further studies examining the effects on the LLE by various characteristics of the worms, most notably age, must be performed to understand variations in dynamical invariants.

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Stability of the stationary solutions of the parametrically driven and damped nonlinear Dirac equation

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The stabilization of damped nonlinear Dirac solitons is addressed by considering a time-periodic parametric force that supplies the required balance between energy losses and gains. A remarkable finding is that we specifically obtain two exact stationary soliton solutions and derive the sufficient conditions for their existence. Their stability is studied by using a variational approach with only two degrees of freedom, which reduces the original problem to an autonomous dynamical system with two fixed points corresponding to the two stationary solutions. It is shown that one of these solutions is stable, whereas the other is unstable. In addition to a stability diagram in the parameter space, we analyze phase portraits that provide a global and detailed view of the dynamics around the stationary solutions. These predictions are revisited linearizing the NLD equation around the exact solution and solving the corresponding Sturm-Liouville problem.

This work has been done in collaboration with Dr. Bernardo Sánchez, Dr. Renato Alvarez and Rafael Jimenez (University of Seville)

Stabilization of Cycles with Stochastic Control

Alexandra Rodkina

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We stabilize a prescribed cycle or an equilibrium of the difference equation with the help of pulse stochastic controls. Our technique, inspired by the Kolmogorov's Law of Large Numbers, activates a stabilizing effect of noise and allows to construct controls for a much wider interval for parameters of an equation, compared to a deterministic control. Our main results are applied to Prediction-Based and Target-Oriented Controls.

Keywords: stochastic difference equations; state dependent noise, stabilization of cycles; Prediction-Based Control, Target-Oriented Control.

Bifurcation, Chaos and Fractals in Dynamics of Certain Special Kinds of Functions

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Over recent years, bifurcation, chaos and fractals are new tools developed for solving more advanced scientific problems. Due to wide-ranging applications, these are increasingly being involved in all areas of research. Many researches show that chaotic phenomena is completely deterministic and characteristic for typical nonlinear systems. The real problems and issues on the frontiers of modern scientific, technological, economical, and social researches are nonlinear in nature. In a nonlinear system, a small change in a parameter can lead to sudden and dramatic changes in both the qualitative and quantitative behaviour of the system. Most nonlinear systems are extremely difficult to solve analytically or much harder to analyze. During the last three decades of the 20th century, the excessive studies of nonlinear dynamics show that chaos occurs widely. Chaos is a phenomenon that has an underlying deterministic rules behind irregular appearances. Nonlinearity is a necessary condition for chaos so that chaotic behavior may get in almost all nonlinear systems. It can visualize effectively through fractals. Recently, chaos has been popular topics of exploration from mathematicians, physicians and scientists. Resulting from this reason, the future of chaos theory is very bright. The purpose of this presentation gives an overview of bifurcation,

chaos and fractals as well as explore bifurcation, chaos and fractals in dynamics of certain special kinds of functions.

Keywords: Bifurcation, chaos, fractals, nonlinear dynamical systems.

Applications of Higher Variational Equations to the Study of Integrability in Dynamical Systems

Sergi Simon

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The question of how Algebra can be used to solve dynamical systems and characterize chaos was first posed in a fertile mathematical context by Ziglin, Morales, Ramis and Simó using differential Galois theory. Their study was aimed at first-order, later higher-order, variational equations of Hamiltonian systems. Recent work by this speaker formalized a compact yet comprehensive expression of higher-order variationals as one infinite linear system, thereby simplifying the approach. More importantly, the dual of this linear system contains all information relevant to first integrals, regardless of whether the original system is Hamiltonian. This applicability to formal calculation of conserved quantities will be the centerpiece of our talk, following an introduction to the requisite context. Important examples will be discussed.

Keywords: Dynamical integrability, chaos, variational equations, first integrals, formal calculus, multilinear algebra, Hamiltonian integrability, functions of several complex variables, meromorphic functions, differential Galois group, monodromy matrix.

Meta-Theories and Scientific Reformation

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Modern science intensively develops various (pseudo-) universal meta-theories, consistently opening fundamentally new perspectives in many areas of knowledge in comparison with the gradually exhausting classical traditional approaches. However, even the simplest review points to their limitations and incomplete effectiveness, due to the systemic

shortcomings of the modern general scientific paradigm, which undoubtedly must be replaced by more advanced developmental paradigms. 1) If historically the first ancient categorical-phenomenal paradigm gave birth to science, then 2) the modern classical (axiomatic, dogmatic) paradigm has improved it in certain particular subject areas, and 3) the next expected universal (met-) paradigm will presumably unite all knowledge into a single system 4) by increasing the level of abstraction until the achievement of a single initial Universal Axiom and meta-concepts derived from it, 5) which should be adequate for all phenomena and 6) form the ultimate Universal meta-theory, 7) resolving all the problems of scientific knowledge. 8) The presented work summarizes the authors' many years of research on this topic and 9) systematically sets out the main provisions of the meta-universalization of knowledge, 10) substantiates the theoretical and practical possibility of obtaining a single universal meta-formalism of the Universe and derivatives of meta-formalisms of phenomena, 11) describing all things let very complex, but a single universal formula. 12) The results of the work have been successfully applied to solve many chronic conceptual problems and 13) have a universal perspective in all areas. 14) Despite the initial state of the Universal Theory, 15) any of its applications are already radically changing the traditional ideas about the world around them, 16) especially complex chronically unknowable phenomena. 17) The ultimate meta-level of abstraction of universal concepts today is a necessary condition for the radical development of knowledge, 18) in which, undoubtedly, many teams of leading scientists should participate, like the scientific school of Bourbaki.

Keywords: Meta-theory, Modern science, Universal Theory, Harmony, Universal Cosmology, Multi-phase Universe, “Boiling” Universe hypothesis.

Contributed and Invited Talks

The Higgs boson and the Higgs field in fractal models of the Universe: supermassive black holes, relativistic jets, solar coronal holes, active microobjects

Valeriy S. Abramov

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To describe the masses of black holes, their relationships with the parameters of the Higgs boson, models based on the distribution density functions of the number of quanta in the ground and excited states for relic photons, and on the basis of the density distribution functions of the radiation intensity are proposed. It is proposed to represent the central region of a supermassive black hole near the upper mass boundary as a Bose condensate from black holes. Various states for a black hole with an intermediate mass are introduced. The following estimates have been made: masses for light black holes, binary and supermassive black holes; the speeds of motion of relativistic jets (emissions of matter); widths of active regions of coronal holes on the Sun; a number of parameters of active microobjects. These estimates are consistent with experimental data.

Keywords: supermassive black holes, Bose condensate from black holes, Higgs boson, relic photons, radiation intensity, relativistic jets, coronal holes on the Sun, active microobjects.

Memory cell based on qubit states and its control in a model fractal coupled structure

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Model fractal coupled structures are considered, for which a characteristic feature of the behavior of the deformation field is the presence of such superposition qubit states where there is no damping. Such states can be memory cell. The possibility of internal and external control of the structure of the memory cell, the possibility of performing the operations of write and delete information has been established. It is shown, that changes in deformation fields in a memory cell are anisotropic. External control of a memory cell in a coupled structure is performed using different fractal indices of separate structures. In this case, fractal indices do not depend on iterative processes. It is shown, that there is a critical value of the fractal

index of separate structures, when passing through which effective damping occurs. This effect can be used to control the storage of information in a memory cell. When fractal indices depend on an iterative process, self-organization (internal control) occurs. By the example of the sinusoidal law of change in the fractal index of separate structures, it is shown, that structures of the following type arise: vertical, horizontal, inclined stripes; lattice structures of various orientations.

Keywords: coupled fractal structures, memory cell, superposition of qubit states, deformation field, control of memory cell structure.

Remark on Stochastic Resonance in the Bullard Dynamo

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At present great attention is paid to the numerically-analytical study of stochastic resonance in systems of various nature (see for instance [1] and references therein). Nevertheless nowadays, as well as in the times of Andronov, Mandelstam and Papaleksi, analog methods for modeling nonlinear phenomena have not lost their significance. That is why in the report we discuss behaviour of one-disk dynamo [2] under the action of

harmonic and random signals in order to use analogous methods for investigation of stochastic resonance in this device. The aim of this research work is to evaluate the effect of such combined signal on the system under consideration in the framework of the linearized Bullard equations. As random signals with zero average the Gaussian delta-correlated noise and the Langevin stochastic process have been used. Estimations of separated influences of harmonic and random external voltage on the linearized Bullard system have been presented. In particular, as physical values characterizing these influences both spectral densities of observables and its bispectra [3] have been calculated. This information is important for layout of the Bullard dynamo to perform analogous research of stochastic resonance in the device in nonlinear regime.

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Keywords: Saddle, Focus, Transient Response, Correlation Functions, Triple Correlation Functions, the Wiener-Khinchin Theorem.

Model of AI Involving Dynamics Symmetry Breaking in Multifractal Medium

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In this paper, we show that the existence of a multifractal medium implies a series of characteristics that are specific to artificial intelligence such as memory, multivalent logic etc. This medium contains the implicit information, the explicit information being evidenced only by a spontaneous breaking symmetry mechanism. In this context, we propose a universal holographic mechanism, mathematically based on group invariances of S (2R) type, through which the transition of implicit-explicit information is realized. The principles discussed are found in the most complex structure from an informational point of view, that is, the human brain.

Keywords: Multifractal medium, Implicit information, Explicit information, Spontaneous symmetry breaking, Brain.

Entanglement distribution in a nitrogen doped graphene nanoribbon

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The concentration and distribution of quantum entanglement is an essential ingredient in emerging quantum information technologies. Here, we focus on the distribution of entanglement in a graphene nanoribbon, a two-dimensional honeycomb lattice of carbon atoms. Graphene, having a perfect two-dimensional crystal structure, has many excellent features. However, disordering through chemical doping with substitutional foreign atoms, such as nitrogen, is a widely used approach to tune the electrical properties of graphene. Accordingly, we turn our attention on a nitrogen doped graphene nanoribbon with a special emphasis on the effect of the concentration of disorder. We try to establish a link between the concepts of quantum entanglement and quantum chaos theories.

Keywords: Graphen, Entanglement, Quantum chaos.

Modeling the economic impact of COVID-19

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We suggest a novel theoretical model to investigate the role of pandemic risk factors in financial markets. To this end, we adopt an incomplete market framework under the assumption that the parameters of the model depend on a stochastic external economic/risk factor, which is assumed to follow a geometric Brownian motion with exponential growth. Based on the theoretical model proposed, we provide an original empirical application to assess the impact of COVID-19 disease on the US stock market using data for the period spanning from January 21, 2020, to December 09, 2020. The results document a negative association between COVID-19 growth rates and US stock market returns.

JEL classification: G17; Q02; Q47

Keywords: COVID-19; Economic impact; Modeling; Stock market

Covid-19 transmission model under delays in the vaccination campaign

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This article aims to model and analyze the spread of the Covid-19 infection under the existence of delays in the vaccination campaign. The proposed model contemplates a dynamic model of three non-linear differential equations where the impact of lag in the vaccination is included. The analysis sees the study of this effect on the dynamics of the equilibrium

points model, conditions of stability, and the existence of oscillations. The analysis addresses an interpretation in terms of public health indicators.

Keywords: Covid-19, delay, nonlinear dynamics, oscillations, chaos.

Wavy Aspects in a Precessing Open Cylindrical Channel

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A series of experiments were carried out to track the forced oscillations in a precessed rotated open cylindrical channel. In such case and as long as the forcing frequency smaller than twice the natural one, a series of inertial waves of the same frequency and different wave lengths will appear. In this paper we solved the problem under irrotational and inviscid conditions in order to find the modes of those inertial waves and to find the resonant modes. The resonant cases happen when the driving frequency comes close to any of the normal or natural one, this will appear as one big progressing wave rotating about the outer circumference of the channel. The observations of such wave accord with small nutation angle about 1° , and it showed that the free surface is disturbed, and turbulent motion clearly noticed, and in some cases the wave breaks down immediately after one revolution. Three different control parameters were changed during the experiments are: nutation angle, rotation rate, and water depth in the channel. Although such oscillations are the expected type to be observed, however other important and interesting phenomenon was noticed clearly, as we deal with open channel not confined filled cylinder, which is the bore or the hydraulic jump almost in all cases, experimentally we tried to track the different types of bore and their distribution in terms of the control parameters and we tried to classify them depending on the theoretical classical classifications under normal conditions it was found that these classifications are right under forced precession conditions.

Quantification of the chaotic phenomenon in natural convection

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In this work a numerical study of unsteady natural convection in a closed square cavity filled with a water copper nanofluid is performed in order to analyze the effects of volume fraction nanoparticles on quantification of phenomenon chaotic. The horizontal walls of this cavity are adiabatic while and the vertical walls consist of two parts of same size; the upper part is hot and the other part is cold. Transfer equations based on vorticity-current-function formulation are solved using finite differences schemes and the implicit alternating direction (ADI) method. We analyze the effects of volume fraction of the nanofluid ranging from 0 to 0.2 on the quantification of chaos. The results are presented by the fractal dimension of attractors, by Lyapunov exponent and by the Kolmogorov entropy. We have remarked that the presence of suspended nanoparticles inside the base fluid causes the delay of the transition.

Keywords: Natural convection, Attractors, Fractal dimension, Lyapunov exponent, Kolmogorov entropy.

Uneven Demographic Dynamics in Nonhomogeneous Economic Communities as an Institutional Trap

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In our work we discuss the demographic dynamics modeling in communities of countries with different levels of economic development. The proposed approach is based on the stratum model of population growth, proposed by the authors earlier. The observed processes of depopulation of the periphery of such communities were studied within the framework of the model. The phenomenon of institutional trap is considered as an explanatory principle of the functioning of complex socio-economic structures. Its main traits are discussed. Based on the proposed model, the forecasts of population growth in several EU countries were calculated and estimates of the parameters of the model were given, at which the institutional trap would significantly affect the forecast. Within the proposed model of institutional trap a set of measures to overcome the negative demographic trends were formulated.

Keywords: Simulation, Demographics, Institutional Trap, Modeling.

Multistability formation and transition to chaos through the bifurcations of two- and three-frequency quasiperiodic oscillations in ring self-oscillating systems based on the van der Pol oscillator

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In this work we consider a ring self-oscillating system composed from a nonlinear amplifier and a chain of linear oscillators. We obtain the dynamical system equations as well as the reduced equations for amplitudes and phases and present the results of numerical simulation and bifurcation analysis. We show that the addition of even one linear oscillator into the feedback loop of the classical van der Pol oscillator leads to emergence of quasiperiodic and chaotic oscillations. In a ring oscillator with three linear oscillators the variation of control parameters leads to

emergence of two- and three-frequency quasiperiodic oscillations as well as transition to chaos through the torus destruction. We demonstrate the appearance of multistable regimes and reveal the bifurcation mechanism underlying the multistability formation. This bifurcation mechanism is based on two consecutive supercritical Andronov-Hopf bifurcations of an equilibrium state and subcritical Neimark-Sacker bifurcation of a saddle limit cycle. The evolution of each coexisting stable regime versus the variation of two control parameters is considered in detail.

Keywords: quasiperiodicity, bifurcation, chaos, van der Pol oscillator, ring oscillator

Modified Blum Blum Shub Random Number Generator using Chaotic Seed Values

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Abstract: One of the exciting application areas of chaotic systems is the generation of random numbers. Chaotic systems defined by expressions such as disorder in order or order in disorder; It has caught the attention of researchers because of their potential to provide unpredictable behavior, which is one of the basic requirements of random number generators. In this study, it is aimed to improve Blum Blum Shub random number generator to a more robust structure for cryptographic purposes by using chaotic seed values. With the proposed hybrid architecture, both an unpredictable structure is provided and the chi-square test has shown that random outputs have good statistical properties. It is thought that the proposed architecture having successful experimental results will have the potential to be used in various applications in the future.

Keywords: chaos, cryptography, random number generator, Blum Blum Shub generator, chi-square test.

Anomalous diffusion through micropillar arrays of suspending microparticles

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We study the diffusion of microparticles suspended in a fluid in micropillar periodic arrays. When the fluid is subjected to small periodic oscillations the particle diffusion can become anomalous. In order to understand this phenomenon, we study the deterministic trajectories without thermal fluctuations. The trajectories of inertial particles are studied using time integration, bifurcations analysis and path-following of periodic orbits. A complex phase diagram of trajectories is revealed. In particular, the trajectories can be chaotic and unbounded. By adding fluctuations, the particles may display super-diffuse or intermittent behaviours especially near the chaotic transitions due to synchronisation phenomena. We point out that the presence of strange attractors is responsible of the anomalous diffusion.

Keywords: Diffusion, Chaotic dynamics, intermittency, Synchronization, Bifurcations, Path-following, suspended particles.

Vivid Chaotic Solitons and Orbital Structures from Discrete Rotation-Translation Sequences in the Plane

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Consider an ordered sequence of operations given by a rotation, a translation, and a reflection. Repeating this sequence operating on an initial discrete point cloud provides for characteristic spatial density patterns in the plane. A wide variety of interesting chaotic point clouds emerge if the rotation angle is a monotonic decreasing function of distance from a rotation singularity. In this way C. Skiadas generated complex structures resembling familiar chaotic flow patterns. Adding a periodic

boundary condition, wavelike patterns can emerge (B. Binder) on the sphere and in the plane. In this talk we focus on conditions, where the point cloud evolves toward an orbital structure or soliton. Starting with the radial Skiasdas basic rotation – translation – reflection density pattern generated by discrete jumps including a radial limit or jumpback condition, we rotate the basic Skiasdas pattern and generate in this way chaotic solitons circular in 2d that show a lot of internal dynamic orbital structures, see below (evolution sequence). We will show with high performance real time simulations or videos the vivid chaotic evolution and dynamics (the live cycles of orbitals) with characteristic numbers or stability conditions. Stable and unstable solitons emerges without any “hard” additional boundary or jumpcondition. They often show a ring shape with several mixed orbits in a kind of hydrodynamic-type orbital flow and an empty region or hole at the center. It exists in a stable form only if the extra rotation of the basic pattern starts very fast and becomes very fast, otherwise the outer radius remains unstable and expands to infinity. The emerging solitons or wavelets can always be characterized by

- orbital wave numbers,
- characteristic radial and orbital density or probability functions,
- geometric phase conditions.

Finally, we propose that this computer experiments show some relevance to quantum physical systems since we have

- a wave attractor showing quantization effects in terms of rotational units,
- a basic non-zero quantum spin in the two main operators (rotation-translation),
- point-like local events with emerging global wave-like probabilistic patterns.

New discrete chaotic cipher key generation for digital embedded crypto-systems

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To benefit of embedded systems which are highly customized providing architectures to suit real-time computing, optimized unit size and low power consumption require highest levels of data communication security which are very useful advantage for telecommunications and networks and Internet of Things (IoT) communication applications that handle sensitive information. This paper proposes a new 3-dimensional discrete time chaos system to generate a robust chaotic cipher data stream to ensure encryption application for secure communication. Dynamical behaviors and security analysis are investigated and compared to current discrete chaotic maps proving its suitability for embedded data encryption systems. Field-Programmable Gate Array (FPGA) implementation design shows better performance and good security robustness compared with previous works while proving the performance improvement of the proposed cipher block in terms of throughput, used hardware logic resources, and resistance against most cryptanalysis attacks.

Keywords: Chaotic generator, Secure communication, Key space, Encryption, FPGA implementation.

A survey on chaos-based cryptosystem: Implementations and applications

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Chaos theory is considered as a tool for studying the systems that show divergence and disorder. After having used discrete mathematics to deduce non-convergence situations, these theories are modeled in the form of a dynamic system and are applied in several domains such as electronic, mechanic, network security...etc. In network security domain, the development of new

crypto-systems based on chaos is a relatively new area of research and is increasingly relevant. The essence of the theoretical and practical efforts in this field derive from the fact that these cryptosystems are faster than conventional methods, while ensuring performance of security, at least similar. In this paper, we discuss several proposals about chaos-based cryptosystem and pseudo-random number generator (PRNG). Moreover, topology and architecture of the proposed chaos systems are detailed. Finally, in order to show the more suitable system for encryption and secure communication, a synthesis comparison is presented and considered.

Keywords: Chaos, Network security, crypto-systems, Communication, PRNG.

Approximate Methods for Solving Hypersingular Integral Equations on Fractals

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The work is devoted to the construction and justification of approximate methods for solving hypersingular integral equations on fractals. The interest in this problem is caused by two circumstances. First, at present, the methods of fractal geometry have become firmly established in many branches of physics and technology, including microwave technologies. Fractals are especially widely used in modeling and designing antennas [1]. Secondly, the branch of mathematics most widely used in antenna modeling is singular and hypersingular integral equations of the first kind. The analytical solutions of these equations are known only in exceptional cases, and therefore the development of numerical methods is required. A large number of publications are devoted to an approximate method for solving singular and hypersingular integral equations on smooth and piecewise smooth curves and surfaces, review of which are contained in [2], [3], [4]. Research on analytical and numerical methods for solving singular and hypersingular integral equations on fractals is just beginning. To date, approximate methods for solving hypersingular integral equations on prefractals of Cantor's set and Sierpinski's carpet have been constructed and

substantiated [5]. In this work, we study one-dimensional hypersingular integral equations defined on fractals with various topologies. Classes of functions, to which the solutions of hypersingular integral equations with different coefficients and right-hand sides are belong, are defined and investigated. Projection-iterative and spline-collocation methods for solving hypersingular integral equations on prefractals and fractals are constructed. The error arising from the approximation of a fractal by a prefractal is estimated.

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Keywords: Fractal Geometry, Prefractals, Topology, Antennas, Integral Equations of the First Kind, Numerical Methods.

Approximate Solution of Inverse Problems of Gravity Exploration on Fractals

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The work is devoted to the approximate solution of inverse problems of gravity exploration on bodies with a fractal structure. It is known [1] that in order to construct mathematical models adequate to the geological reality, it is necessary to take into account the orderliness inherent in geological environments. One of the manifestations of orderliness is self-similarity, which

remains during the transition from the microlevel to macrolevel. Scaling of geological media can be traced in petrophysical data and in anomalous fields. It should be noted that in real structures there is no infinite self-similarity and scaling must be considered in a certain range. The work investigates analytical and numerical methods for solving inverse contact problems of the logarithmic and Newtonian potential in the generalized setting [2]. In the case of a Newtonian potential, the problem is formulated as follows. It is required, having three independent functionals of the gravity field above the Earth's surface and additional information on the self-similarity of the disturbing body, to determine the depth H , the density $\sigma(x,y)$ and the surface $H-\varphi(x,y)$ of the perturbing body, occupying the region $H \leq z(x,y) \leq H - \varphi(x,y)$. In this work, when constructing fractal models of geological environments, the authors proceed not from fractals, but, following [3], from additions to fractals, since areas (volumes) of additions tend to areas (volumes) of the original body. On the basis of the apparatus of fractional measure and fractional dimension [4], the processing of disturbances of the Earth's gravitational field is carried out. Taking into account the fractal components of gravitational fields makes it possible to clarify the structure of the disturbing bodies.

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Keywords: Self-similarity, Fractional Measure, Disturbances of the Earth's Gravitational Field, the Logarithmic Potential, the Newtonian Potential, Petrophysical Data.

Two-dimensional chimera states and their synchronization

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The chimera state formation is the new phenomenon discovered in the beginning of the century. Firstly these structures were obtained in the one-dimensional chains with periodic boundary conditions. Then, two-dimensional chimera structures based on spiral waves were revealed. In this work we show new chimera structures which are based on target waves. Besides, we study spiral wave chimera states in detail, and compare them with target wave chimera states. Finally, we investigate the synchronization phenomenon for both target and spiral wave chimera states and show differences in their nature based on the features of their synchronization.

Keywords: spiral wave, target wave, chimera state, synchronization.

The Parrondo's paradox for tent maps

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The dynamic Parrondo's paradox states that the combination of two dynamically simple (resp. chaotic) maps gives rise to a chaotic (resp. simple) dynamical behavior. Here, we analyze whether the paradox appears when we combine piecewise linear maps. In particular, we consider the combination of a piecewise linear homeomorphism and the family of tent maps $a \cdot x \cdot \min\{x, 1-x\}$, where $0 < a < 1$ and check whether the paradox simple+simple=complex appears. On the other hand, we consider two tent maps and check whether the paradox comple+complex=simple takes place.

Keywords: Parrondo's paradox, tent maps, piecewise linear maps, topological entropy, chaoticity, regular dynamics.

A note on combining chaotic dynamical systems using the fuzzy logic XOR operator

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In this paper we explore whatever combining two chaotic dynamical systems using the fuzzy logic operator XOR can maintain or not the chaotic properties of the resulting dynamical system. This study is motivated by techniques used in applications to secure communications images encryption and cryptography.

Keywords: Chaos, fuzzy logic, ergodic theory, full branch.

Detecting Causal Relations between Time Series with the Cross Markov Matrix Technique

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A new measure for the characterization of interconnected dynamical systems coupling is proposed. The proposed measure, called Cross Markov Matrix, starts from the idea introduced by Campanhao et al (PLoS ONE 6, e23378 (2011)) of transforming time series into complex networks based on the transition probability in a Markov model. Our approach extends this idea to coupled time series by mean of an interleaving technique which allows the construction of a complex network from a pair of time series. The topological structure of the complex network changes with the strength of the coupling between the time series, the coupling strength. The topological changes may be assessed by monitoring the entropy of the adjacency matrix, represented as an image or by mean of the other indicators as, for example, the medium articulation. The properties of the Cross Markov Matrix have been investigated with the help of a systematic series of numerical tests using synthetic data. The potential of the approach is then substantiated by the analysis of various real-life examples, ranging from environmental and global climate

problems to the mutual influence between media coverage of Brexit and the pound-euro exchange

Keywords: Causality Markov Matrix, Time series, Complex networks, ENSO, Brexit.

Addiction during COVID-19

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The COVID-19 pandemic has disrupted the global economy, society, psychology, and behavior, as well. The fear of the COVID-19 spread and the oppression due to the lockdown measures had a negative impact on human behavior, leading to addictive behaviors, as stress-relievers. In this work, through a financial framework, we examine the causal and long-run effect of the COVID-19 pandemic on some of the biggest companies' stocks related to gaming, online gambling, liquor and tobacco, using time series and multifractal analysis. Based on our findings, the COVID-19 confirmed cases exhibit a positive long-run relationship with the examined stocks and this relationship has rich multifractal properties with small changes being more persistent than large ones, as shown by the multifractal detrended cross-correlation analysis.

Keywords: COVID-19; gaming; gambling; alcoholism; tobacco.

The electromagnetic interaction among watery precipitations in the atmosphere

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When a raindrop, mist or dew falls through a thundercloud, it is subject to strong electromagnetic fields that pull and tug on the droplet, as well as a soap bubble in the wind. If the electric field and consequently the

electromagnetic effect is strong enough, it can cause the droplet to burst apart, creating a fine, electrified mist. Droplets tend to form as perfect little spheres due to surface tension, the cohesive force that binds water molecules at a droplet's surface and pulls the molecules inward. The droplet may distort from its spherical shape in the presence of other forces, such as the force from an electric field. Sometimes, we have anomalous diffraction because of the different shapes of the droplets. While surface tension acts to hold a droplet together, the electric field acts as an opposing force, pulling outward on the droplet as charge builds on its surface.

Interaction of a magnetic field with a charge object

How does the magnetic field interact with a charged object? If the charge is at rest, there is no interaction. If the charge moves, however, it is subjected to a force, the size of which increases in direct proportion with the [velocity](#) of the charge. The force has a direction that is perpendicular both to the direction of motion of the charge and to the direction of the magnetic field. There are two possible precisely opposite directions for such a force for a given direction of motion, extremely in cases of storm, followed by lightning and thunder. Certainly, the cold thermal currents contribute to the condensation of water vapor and the creation of various water sediments. . This apparent [ambiguity](#) is resolved by the fact that one of the two directions applies to the force on a moving positive charge while the other direction applies to the force on a moving negative charge. Figure 1 illustrates the directions of the magnetic force on positive charges and on negative charges as they move in a magnetic field that is perpendicular to the motion. Depending on the initial orientation of the particle velocity to the magnetic field, charges having a constant speed in a uniform magnetic field will follow a circular or helical path.

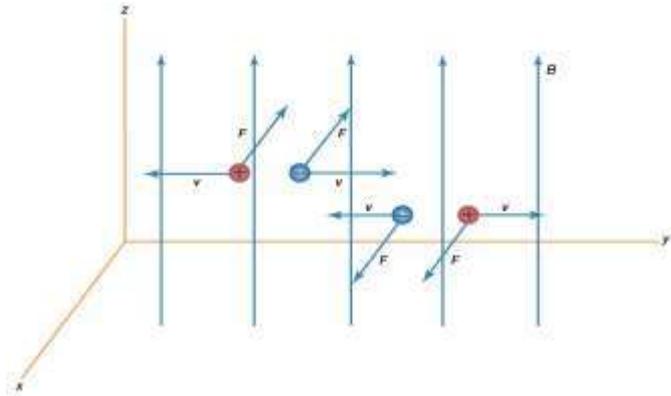


Figure 1.

Keywords: Watery precipitators, electromagnetic field, atmospheric droplets.

Understanding shifts in intra-firm trade portfolios through economic system entanglement

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In the last five years, Italy has seen a noticeable and steady increase in the supply of trade credit, granting of extensions, and general systemic business-to-business financial support. Focusing on system entanglement, this paper examines the impact in Italy of bank valuations of creditworthiness and credit intermediation on intra-firm trade portfolio dynamics. We further consider the impacts of exogenous shocks to the economy and other disruptive events on payment regularity and risks of insolvency in intra-firm transactions. Mapping portfolio dynamics to a quantum super-system with a Hamiltonian space of phases, we demonstrate that the performance of intra-firm portfolios depends concurrently on bank valuations and that system entanglement allows us to examine the extent to which economic disruptions shift portfolio dynamics from their state of equilibrium.

Keywords: System entanglement; intra-firm trade; portfolio dynamics; credit valuation

JEL Codes C02; C61; G24; H12; M21

Levels of local chaos for special Blocks Families and applications for Turing Machine

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In this work we going to study the properties of some chaotic properties via Furstenberg families, specially using other levels of Blocks families. Futhermore, We are going to relate the Li Yorke and Distribution chaos levels through the existence of certain categories of block families, which contain IP families and Weakly Thick families. Then, we are going to demonstrate that there is equivalence between chaotic localities in time actions that are closed under addition and multiplication, showing at the end some applications in types of shift systems from an ergodic point of view. At the end, we show some results for dynamics that describe turing machines and we study equivalences using the aforementioned families.

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Analytcs of Portfolio Selection Dimensions: Return, Risk, Diversification

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The modern portfolio theory approach in portfolio selection, given the return of some financial assets, is risk minimization measured by the variance of associated portfolio. The associated return-variance efficient frontier represents the available solutions. The Sharpe Index (return/risk) is the appropriate portfolio selection criterion to balance the dimensions of return and risk. The known problem of this approach is the portfolio diversification, the concentration of optimum portfolio on few of the assets. The diversification which is a third dimension of portfolio can be measured by the entropy of portfolio asset weights. A return-entropy efficient frontier used where the maximization of portfolio weights entropy is the objective of optimization and naïve solution is the equal weights portfolio. In the return-entropy efficient frontier Entropic Return index (returnxentropy) is the proposed criterion to overcome the naïve equal weights solution. The assessment of those two portfolio efficient frontiers is the purpose of this paper. Using a dataset of 10 stocks the optimum portfolios, by the two efficient frontiers and related indexes, evaluated in the out of sample period performance. The observed performance differences, in short and medium term out of sample horizon, indicates that for the portfolio selection problem all the three dimensions are important.

Keywords: Maximum Entropy Frontier, Portfolio Selection, Equal Weights Portfolio, Portfolio Diversification.

Detection of Early Warning Signals for Self-

Organized Criticality in Cellular Automata

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Detection the precursors of critical transitions in complex systems is one of the most difficult and still unsolved problems. This problem has not received a final solution, not only for real complex systems, but also for model systems capable to self-organize into the critical state. The presented paper is devoted to early detection of time moments of selforganized critical transitions in cellular automata as a result of the analysis of the time series they generate for a number of grains falling from the grid. It was found that cumulative moments of probability distribution and cumulative scaling exponents are quite informative indicators for early detection of critical transitions. General features of the behavior of indicators when approaching a critical point are established for the time series generated by cellular automata with different rules.

Keywords: Cellular Automata, Sandpile Model, Self-Organized Criticality, Time Series, Probability Moments, Multifractality.

Double Symmetry and Generalized Intermittency in Transitions to Chaos in Electroelastic Systems

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Mathematical models of a deterministic system of the type "analog generator-piezoelectric transducer" are considered. A double symmetry, atypical for dynamical systems, is found in the alternation of scenarios of transitions from regular attractors to chaotic ones. Typical for dynamical systems is the behavior when, with an increase in the value of the bifurcation parameter, the following chain of transitions to chaos is observed: a cascade of bifurcations of doubling the period of the limit cycle - a chaotic attractor - a periodicity window, then again a cascade of

bifurcations of doubling the period of a limiting cycle - a chaotic attractor - a window of periodicity, and so on. Accordingly, with a decrease in the value of the bifurcation parameter, a chain is observed: chaos-window of periodicity-intermittency-chaos. In (1), a situation was described in which there is a certain "median" value of the bifurcation parameter relative to which, both with its decrease and with its increase, one of the above-described chains of transitions to chaos is observed. That is, there is some symmetry in the alternation of scenarios of transitions to chaos. For the considered system was found the symmetry inside symmetry: the described above chains of scenarios is located at the "median" point of other wider symmetric chains of transition to chaos. Also, for the first time for the considered system, a transition "chaotic attractor of one type-chaotic attractor of another type" through generalized intermittency was discovered (2). One of the distinctive features of such a transition is the appearance of coarse-grained (rough) laminar phase instead of laminar phase of usual intermittency.

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Keywords: nonideal electro-elastic systems, scenarios of transition to chaos, generalized intermittency.

Doubochinski's Argumental Macro-quantic Oscillator

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The argumental macro-quantic Doubochinski's oscillator in classical physics represents a quantum analogue of harmonic oscillator in quantum mechanics. In this case, it is not the acting forces that are considered, but their interaction with each other, when the oscillator performs a frequency-phase modulation of a periodic force acting on it. Namely, the oscillator "decomposes" this force into a series and "selects" the one component of the decomposition spectrum that is equal or is close to the oscillator's

natural frequency. The presence of a number of spectral components enables the oscillator to implement a discrete series of stable quantized amplitudes, which are determined by the initial conditions.

Keywords: nonlinear waves.

Bivariate fractal interpolation

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Fractal interpolation functions that can create smooth and non-smooth interpolants constitute an advancement to classical or traditional interpolants. We try to construct a bivariate fractal interpolant by using a suitable iterated function system. Our construction of bivariate smooth fractal functions permits us to seed shape parameters in the functional equation of fractal interpolants.

Estimating Lyapunov exponents by local jacobian indirect methods

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This paper presents a summary of the main concepts and definitions related to two new local jacobian indirect methods for estimating the Lyapunov exponents from time-series data. We provide a discussion about the right procedure to obtain the partial derivatives based on the

local jacobian approach. The main differences with the global jacobian approach are also highlighted. In addition, it illustrates how to get a consistent estimator of the Lyapunov exponent based on the local jacobian approach. Then we describe the key features about the local polynomial kernel method by approximating the unknown nonlinear system through a local polynomial of order greater than one. We also provide the main details about the local neural net method by approximating the unknown nonlinear system through a feed-forward single hidden layer local neural net. Finally, we report the empirical results of this paper and some concluding remarks.

Keywords: Chaotic time series, Lyapunov exponents, Local Jacobian indirect methods, Polynomial kernel models, Neural net kernel models.

Hidden Bifurcations and the Longitudinal Expansion for the Size of the x-Projection of the Scrolls via Parallel Transformation

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In this paper, a study hidden bifurcation of multi scroll chaotic attractors via parallel transformation is proposed. The verification of such a hidden bifurcation on two parameters corresponds to the procedure presented by Menacer, et al. (2016) for Chua multi scroll attractors. These hidden bifurcations (the number of the spiral) are governed by an additional homotopy parameter ε which is introduced to unfold the real structure of the multispiral attractor, instead of the discrete parameter N which is considered constant now. Also, the aforementioned method was employed again in the research to show the effect of parameter ε , by the number of scrolls obtained, on the longitudinal expansion for the size of the x-projection of the scrolls, when the parameter p is varied.

Keywords: Chua system, Hidden bifurcations, Parallel transformation, Multiscroll chaotic attractors.

Hurricanes as Bose-Einstein condensates of atmospheric dynamics

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Although the lifecycle of hurricanes is well understood, it is a struggle to represent their dynamics in numerical models, under both present and future climates. We consider the atmospheric circulation as a chaotic dynamical system, and show that the formation of a hurricane corresponds to a reduction of the phase space of the atmospheric dynamics to a low-dimensional state. This behavior is typical of Bose-Einstein condensates. These are states of the matter where all particles have the same dynamical properties. For hurricanes, this corresponds to a "rotational mode" around the eye of the cyclone, with all air parcels effectively behaving as spins oriented in a single direction. This finding paves the way for new parametrisations when simulating hurricanes in numerical climate models.

Multifractal analysis of bioenergy transport in a protein nanomotor

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The potential application of biological molecules as functional devices has been heralded as the dawn of new era in biotechnology and medicine. In this regard, molecular motors have attracted most attention for decades. One of the obstacles for the practical application of molecular devices is

the lack of functional control methods in biological media, under changing conditions. In the current study, we have studied the bioenergy transfer in a protein chain as a self-powered nanomotor. We have used a Hamiltonian model based on Pang model for studying the bioenergy transport in protein chains. The effect of different factors on the energy transfer in protein is studied to obtain the best functional condition for protein machine. The effect of temperature of environment plays the important role in control of energy transfer in system. On the other hand, the external mechanical tension as a vibrator can ! increase the energy flowing in our system. The evolution equations of system are derived and the energy flux is calculated using the Hamiltonian equation. The best range of effected parameters can be investigated using the multifractal analysis. Generalized dimension can distinguish the distinct behavior of system. The chaos theory tools can verify and estimate the results. Generally, one can engineer a self-powered nanomotor based on protein chains and control bioenergy transfer through.

Keywords: Nanomotor, Protein, Biological energy, Energy transfer, Multifractal analysis.

Piezo spintronic effect in DNA molecular Chains

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Recent efforts have been focused to produce nanoscale spintronic systems based on molecular materials. Molecular spintronics is an exciting concept for spin-based quantum computing. Spintronics is the combining the electronic with electron spin which is the attractive field for process and transfer the information. On the other hand, creating the pure spin currents in response to strain can be studied in the content of piezo spintronic effect. In this regard, we have tried to design a DNA based piezo spintronic device. We have proposed a theoretical model for controlling the spin current in DNA based on coupling between mechanical distortions and spin degrees of freedom. We have used a Hamiltonian approach based on the tight-binding model combined with spin degree of freedom. The mechanical tension is applied through a correction on the hopping

term of Hamiltonian. We have used the chaos theory tools! to study the spin transport properties in the system. The evolution equations of system are obtained and the net spin current in the presence of the external stress is derived. The obtained results determine that different DNA sequences show distinct behavior with respect to the mechanical tension. Also, the regions in the parameter values in which the maximum spin current flows through the system can be investigated. The mechanical tension can adjust the spin current flowing through the system. Therefore, one can design and control a novel piezo spintronic nanodevice based on DNA sequences.

Keywords: Piezo spintronic, Mechanical tension, DNA chain, Spin current, Chaos theory tools.

Optimality Principles in Solution of Nonlinear Control Problems under Uncertainty Conditions

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We study the problem of estimating reachable sets of nonlinear dynamical control systems considered under assumption of uncertainty in system parameters and in initial system states. It is assumed that only bounding sets are available for unknown terms, and no additional statistical information is provided on their behavior. Basing on principle results of the theory of trajectory tubes of control systems [1-4] we find solutions for control problems under uncertainty and investigate their properties. The algorithms of constructing ellipsoidal estimates for the solution tubes are discussed and tested. Applications to the problems of behavior of competing firms, population growth models, environmental change, the development of certain competing industries, etc. are discussed.

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- Keywords:** Nonlinear control systems, Estimation problem, Set-membership uncertainty, Ellipsoidal calculus, Maximum principle, HJB equation.

Chaotic driven systems of qbit on KAM surface

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The classical kicked top model plays important role in the qbit systems and the chaotic properties of the entanglement in the quantum mechanics. This driven model can be written by the probability distributions in the phase space considering the statistical complexity. The chaoticity is presented by the strength excitations on the KAM surface of the kicked top and kicked rotor systems. This quantity well characterize the nonlinearity of the driven model of the Hamiltonian qbit systems.

Keywords: Chaotic modeling, qbit, Hamiltonian system, statistical complexity, simulation of the kicked top and kicked rotor model.

Global Bifurcations of Limit Cycles and Chaos Transition in Multi-Parameter Dynamical Systems

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We carry out a global qualitative analysis of multi-parameter polynomial dynamical systems. First, using our new bifurcational and geometric methods, we solve Hilbert's Sixteenth Problem on the maximum number of limit cycles and their distribution for the 2D Liénard and Euler-Lagrange-Liénard polynomial, Holling and Leslie-Gower quartic, and also Kukles cubic-linear dynamical systems. Then, applying a similar approach, we study 3D polynomial systems. In particular, we complete the chaos transition scenario for the classical Lorenz system connecting globally the homoclinic, period-doubling, Andronov-Shilnikov, and period-halving bifurcations of its limit cycles which is related to Smale's Fourteenth Problem. We also carry out a global bifurcation analysis of the 3D Topp model for the dynamics of diabetes.

Keywords: Multi-parameter dynamical system, Bifurcation, Limit cycle, Chaos.

The Atom, from a Mathematical-Physical Perspective

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In this paper, an exhaustive study on the problem of atomicity with respect to set functions is provided. Different types of atoms are discussed, the relationships among them are studied and several examples and physical possible implications and applications are obtained.

Keywords: Atom, Pseudo-atom, Minimal atom, Set function, Self-similarity.

Nonlinear forced vibrations of plates with oscillating inclusions

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These studies are concerned with the forced vibrations of rectangular plate which includes oscillating inclusions uniformly distributed in the carrier elastic medium. Plate's equations of motion consist of the Lamé equation for a carrier material and the equation for oscillators representing the inclusions. The dynamics of oscillators is defined by the nonlinear friction force defined by the modified Coulomb's law, whereas the elastic force is cubic nonlinear. The simply supported forced plate is considered. Using the Galerkin approximation, the problem of plate vibrations is reduced to the study of a system of ODEs. According to numerical and qualitative analysis, application of the harmonic force causes the appearance of periodic, quasiperiodic and chaotic regimes in the system. In particular, the analytical expression is derived for the periodic regimes of the forcing frequency. The scenarios of vibrations development is identified at the variation of forcing amplitude. The quasiperiodic and chaotic modes are studied by means of Poincaré section technique, Fourier and Lyapunov spectra analysis. The statistical properties of the sequences of temporal intervals between maxima of system's solution are considered in more detail. The sequence extracted from the chaotic trajectory is shown to possess long-term correlations and approximately obeys the Weibull distribution.

Keywords: Complex plate, Nonlinear vibrations, Chaos, Tori, Correlations.

Complex Networks and Causality between Time Series

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At present some of the most interesting scientific problems require investigating short, irregular, chaotic and sometimes corrupted time series. Identifying the mutual, causal influences between such signals is particularly challenging, particularly because in many instances interventions and experiments are difficult, expensive or utterly impossible. The conversion of time series into complex networks has recently become a very active area of research. The properties of the networks can be quantified with various tools, typically converting the adjacency map into an image before deploying image processing techniques. The proposed methods are exemplified with real time cases, ranging from atmospheric physics and epidemiology to thermonuclear fusion.

Keywords: Time series, complex networks, coupling, synchronization.

The Forecasting Performance of Nearest Neighbor in Realized Correlation

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The present paper examines the performance of k nearest neighbor in out-of-sample forecasting the realized correlation. Realized correlations are estimated based on realized covariances. Empirical findings concern international stock markets and are consistent in different forecasting horizons. The markets considered are: France, Germany, Brazil, Mexico, Japan, USA, India and China. The conception of nearest neighbor conceptualizes the incremental information of correlations.

Modeling working memory in spiking neuron network accompanied by astrocytes

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We propose a novel biologically plausible computational model of working memory (WM) implemented by the spiking neuron network (SNN) interacting with a network of astrocytes. SNN is modelled by the synaptically coupled Izhikevich neurons with a non-specific architecture connection topology. Astrocytes generating calcium signals are connected by local gap junction diffusive couplings and interact with neurons by chemicals diffused in the extracellular space. Calcium elevations occur in response to the increase of concentration of a neurotransmitter released by spiking neurons when a group of them fire coherently. In turn, gliotransmitters are released by activated astrocytes modulating the strengths of synaptic connections in the corresponding neuronal group. Input information is encoded as two-dimensional patterns of short applied current pulses stimulating neurons. The output is taken from frequencies of transient discharges of corresponding neurons. We show how a set of information patterns with quite significant overlapping areas can be uploaded into the neuron-astrocyte network and stored for several seconds. Information retrieval is organised by the application of a cue pattern representing the one from the memory set distorted by noise. We found that successful retrieval with level of the correlation between recalled pattern and ideal pattern more than 90% is possible for multi-item WM task. Having analysed the dynamical mechanism of WM formation, we discovered that astrocytes operating at a time scale of a dozen of seconds can successfully store traces of neuronal activations

corresponding to information patterns. In the retrieval stage, the astrocytic network selectively modulates synaptic connections in SNN leading to the successful recall. Information and dynamical characteristics of the proposed WM model agrees with classical concepts and other WM models.

Keywords: Spiking Neural Network, Astrocyte, Neuron-Astrocyte Interaction, Working Memory.

Deterministic vs Stochastic Behavior in Bitcoin Dynamics: evidence from Poincaré Recurrence Theorem

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In this paper, starting from the Ergodic Theory, we try to present a model describing the dynamics of the Bitcoin cryptocurrency system. In our model the dynamics of the Bitcoin appears as dual: it mostly behaves as a deterministic system and in some time intervals, much shorter, it enters a stochastic regime. We also focus our study on understanding the transition from one regime to another and we use Poincaré's theorem. We apply our hypothesis to real data and we are able to explain the reason why the Bitcoin system is affected by such a "high volatility".

Keywords: Finance, Bitcoin, Ergodic Theory, Deterministic, Stochastic

Spectral analysis and invariant measure in studying the dynamics of a metabolic process in the glycolysis-gluconeogenesis system

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The causes for the appearance of autooscillatory processes in the chain of metabolic glycolysis-gluconeogenesis reactions were studied, using the constructed mathematical model. The causes can influence an autooscillatory process in the whole metabolic process in a cell.

Using the spectral analysis, we have investigated the influence of the small parameter changing on the self-organization and the metabolic process adaptation to changes in a cell and in the environment.

Keywords: self-organization, strange attractor, glycolysis, gluconeogenesis, Fourier series, invariant measure, bifurcation.

A double-pendulum interacting with an ideal gas: investigation of a mixed chaotic-stochastic dynamics

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An interesting question in nonlinear dynamics concerns how chaotic systems interact with a stochastic environment like a thermal bath. In order to tackle this problem we have numerically investigated the evolution of a Hamiltonian dynamical system, namely the double pendulum, interacting with an ideal gas. The purpose of this investigation is to infer, under the condition of quasi thermodynamic equilibrium, the contributions to the system's evolution of both deterministic chaos and the stochastic perturbations ensuing from the collisions of the gas particles with the double pendulum. The analysis is carried out by applying robust methods of time-delay embedding and chaos detection to scalar time series produced via numerical simulations. The maximum Lyapunov exponent is evaluated by means of the so-called divergence rate method. Preliminary results show that, even in thermal equilibrium conditions, chaos makes up a relevant contribution to the system's dynamics.

Keywords: chaos detection, Hamiltonian chaos, double pendulum, noise, divergence exponent.

Stability of a moving current carrying conductive string in a magnetic field

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A wide range of physical systems may be modeled as axially moving strings such as belts, tapes, wires and fibers with applied fields of electromagnetic origin. Such systems have broad applications in chemical and textile industries or in space applications. Regarding specific applications, the string system can be used for load transportation, like that of space elevators. For example, space elevators could utilize conductive carbon nanotubes which can be treated like an electrically conductive string system with an applied electromagnetic field for control. In this study we model the motion of a current carrying conductive string in a magnetic field through Newton's Law. The Stability of lateral vibrations of a moving current-carrying string in a periodic magnetic field is investigated. It is assumed that the string is moving with a constant velocity between two rings which are at finite distance from each other. Directions of the magnetic field and the motion of the string coincide. The problem is first considered in astatic setting. A critical value of the magnetic field is shown to exist when the uniform (along a string line) motion of a string is unstable. The critical value depends on the speed of motion, applied magnetic field, and current. In the dynamic setting, the condition for stability is also discussed. It is shown that there is a divergence between the critical values in the dynamic and static cases. In a case of piecewise constant periodic magnetic field Floquet dispersion equations are developed, which allows determination of the instability boundaries of string. In a case of nonlinear moving string when a periodic nonstationary current flows through the string we develop approximate solution. Domains of parameters are defined when the string falls into a pre-chaotic state, i.e., the frequency of vibrations is doubled.

Keywords: Stability, dynamics, nonlinear conductive string, magnetic field.

Exploring dynamical regime change in natural and economic systems

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One of the major challenges in forecasting from non-stationary data is to determine whether one has seen any regime change. We shall explore the use of methods of non-linear dynamical systems (for example correlation dimension, recurrence quantification analysis) to test for the occurrence of regime change, with particular emphasis on economics and global climate systems. For example, global average temperature (source: https://www.ncdc.noaa.gov/cag/global/timeseries/globe/land_ocean/yt/12/1880-2020)

Increased 0.00426 ± 0.00045 deg C/year from 1880 to 1980, but 0.01758 ± 0.00128 deg C/year from 1980 to 2019; however, this is an *a posteriori* analysis with a visual estimate of the breakpoint at 1980. In economics we are now seeing long-term interest rates $< 1\%$ (US 10-year bond) and short-term rates near 0, a sharp decline from previous historic levels. Again, does this represent dynamical regime change or a long cycle extending over 10 or more business cycles.

Effects of localized mechanical stimulation of the heart on spiral wave dynamics

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Spiral waves of electrical activity in cardiac tissue are abnormal rhythms that have been largely linked to the onset of lethal cardiac arrhythmias such as Ventricular Fibrillation (VF). Breaking up of these spirals may form multiple chaotic spiral waves, which might lead to cardiac arrest. Studies on the mechanisms leading to spiral breakup is therefore

important in cardiac arrhythmias. In the heart, the electrical waves initiate muscle contraction, through the mechanism of the excitation-contraction coupling, while contraction causes tissue deformation, which in turn feeds back on the wave propagation and affects electrophysiological properties via mechano-electrical feedback (MEF). Many studies have shown the importance of the MEF. It has been, for instance, shown that the mechanical impact on the chest, in the area directly over the heart, can either cause *Commotio Cordis* when the chest receives a blow, or terminate cardiac arrhythmia such as VF using the human fist on the chest during precordial thump. In this work, we study the effects of mechanical stimulus on the dynamics of spiral waves using a biophysically detailed electromechanical model of cardiac tissue. Numerical examples will be provided to illustrate the underlying mechanisms. We show that mechanical stimuli can result in the breakup of spiral waves into turbulent wave patterns or may lead to drifting of a stationary spiral wave, depending on the magnitude and the region where stimuli is applied. **Keywords:** Cardiac arrhythmias, cardiac electromechanical simulation, mechano-electrical feedback, spiral wave chaos, mechanical stimulation, chaotic simulation.

A quantum dynamical map in the creation of optimized chaotic S-box

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A serious challenge in cryptosystems is designing of substitution boxes (S-boxes). S-box, which performs confusion, has been widely employed in traditional cryptographic standards such as the Data Encryption Standard (DES) and the Advanced Encryption Standard (AES) for encryption and decryption process. Creating a new S-box is essential because previous designs do not yet have the highest good S-box criteria. Recently, the use of chaos in the design of efficient S-boxes has been considered. From the mathematical perspective, an $n * m$ S-box is a nonlinear mapping $S: V_n \rightarrow V_m$, where V_n and V_m represent the vector spaces of n, m elements from $G(2)$. Many classic maps have been used to form the chaotic S-box. In this article, after introducing a new quantum system, we examine its effect on the formation of chaotic S-boxes and compare them with chaotic S-boxes based on classical maps. Also, in the previous work, PSO algorithm was improved with the help of classical map and then used in the optimization of chaotic S-boxes. In this work, we examine the impact of the use of the quantum map in improving the PSO algorithm as well as its application in improving the performance of chaotic S-boxes. Then, by changing the type of optimization, we examine its effects. For the first time, the harmony search algorithm is improved by the said quantum map, and then we use it to optimize the produced chaotic S-box. By examining the performance of generated S-boxes by common attacks such as nonlinearity, BIC, SAC, LP, and DP. The results for the improved harmony search algorithm is better. The introduced S-boxes can be used in all image encryption, steganography, watermarking, and quantum digital signatures to increase security.

Keywords: Quantum dynamical map, Substitution box(S-box), Harmony search algorithm, Particle swarm optimization (PSO), Nonlinearity.

Boolean chaos in electronic logic gates

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An investigation of Boolean chaotic behavior in a chaotic circuit system based on an autonomous Boolean network with the application of a logic simulation is presented. The behavior is confirmed by experimental reproduction and we identify some non-ideal features in real logic gates that allow the emergence of chaos in physical networks.

Keywords: logic gates, chaos, Boolean networks.

A Substitution Box Structure Based on Chaotic May Map

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In this study, a cryptographic component is designed based on chaotic systems. The use of chaotic systems in the design of substitution-box structures as cryptological components is a common approach in the literature. The difference of this study from the similar ones is that the chaotic May map as a chaotic system has been used for the first time in the design of the substitution-box structures. The nonlinearity value of the substitution-box structure based on the chaotic May map has been measured as 106.75. This value is expressed as the highest value that can be achieved for chaos-based substitution-box structures. Considering the design criteria, it is thought that this successful substitution-box structure can be used in many information security applications in the future studies.

Keywords: discrete-time chaotic systems, May map, substitution-box, cryptography.

Investigating the presence of chaotic structure in the crude oil market

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This research studies the existence of nonlinear patterns and chaotic nature in crude oil spot price. Using daily data, three classes of tests are applied to detect chaotic behaviour: metric, topological and entropy-based. Tests include those best suited for noisy time series analysis.

Empirical results suggest that both the GARCH/EGARCH models explain a significant part of the nonlinear structure that is found, as well as chaos in crude oil market cannot be assumed. Further research is also needed to examine other types of nonlinear models, both parametric and nonparametric, that can explain the nonlinear structure found in the series more completely. This work contributes to clarifying the behavior of a highly relevant variable in global economy, such as the oil price.

Keywords: Crude, Oil markets, Nonlinear dependence, Dynamical System, Chaos, GARCH modelling.

The interaction of memristor in nonlinear networks for image and signal processing

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In this paper, we describe the application of memristor in the neighborhood connections of cellular nonlinear networks (CNN) essentially for image and signal processing. We focused particularly on the interaction of memristors between two cells allowing us to study the contribution of the memristor qualitatively and quantitatively. The dynamics and the steady state response of each cell is described. The resistance of a memristor is not fixed, hence the study takes into account the initial state of the memristance characterized by the previous amount of charge passed through the memristor. We show that the system transition and the steady state response depend strongly on the history of the memristor.

Keywords: Memristor, cellular nonlinear networks, image processing, system dynamic, steady state response.

Anomalous scaling under the influence of helicity and finite time correlations in the Kazantsev-Kraichnan model of fully developed turbulence

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The influence of the helicity (the spatial parity violation) on the anomalous scaling behavior of a weak magnetic field in the framework of the kinematic Kazantsev-Kraichnan model with finite-time velocity correlations and in the presence of a large-scale anisotropy is investigated using the field theoretic renormalization group approach together with the operator product expansion technique in the two-loop approximation of the corresponding perturbation theory. The relevant composite operators which drive the anomalous scaling of correlation functions are analyzed. The two-loop expressions of the anomalous dimensions of the leading composite operators are found as functions of the spatial dimension d , of the renormalization group fixed point of the parameter u , which characterizes the presence of the finite-time velocity correlations, and of the helicity parameter ρ . The combined effects of helicity and finite time correlations of the velocity field on the anomalous scaling are discussed and compared to previously obtained results for non-helical Kazantsev-Kraichnan model of a passive vector field as well as to Kraichnan model of a passive scalar advection with finite time correlations in the second order approximation.

Keywords: Anomalous scaling, Passive vector advection, Weak magnetic field, Kazantsev-Kraichnan model, Finite time correlations, Spatial parity violation, Helicity, Composite operators, Two-loop approximation, Renormalization group analysis, Fully developed turbulence.

Stabilization of two-dimensional patterns in a weakly-dissipative Bose-Einstein condensate

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The stabilization of two-dimensional patterns by the dissipative confinement potential is demonstrated. Both analytical and numerical analysis based on the generalized dissipative Gross-Pitaevskii equation uses a close analogy between the Bose-Einstein condensate dynamics and the mode-locked fiber laser operating in the anomalous dispersion regime. In the last case, the formation of stable two-dimensional patterns corresponds to spatiotemporal mode-locking using the mode cleaning enhanced by dissipation. The main scenarios of pattern destabilization, varying from soliton dissolution to its splitting and spatiotemporal turbulence, are analyzed depending on the graded dissipation.

Keywords: Solitons in the Bose-Einstein condensates; Solitons in the optical fibers.

Ubiquitous forbidden patterns in codon triplets side chain sequences of SARS-COV-2

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Each amino acid in a polypeptide chain has a distinctive R-group associated with it. We report here a novel method of species characterization based upon the order of these R-group classified amino acids in the linear sequence of the side chains associated with the codon triplets. In an otherwise pseudo-random sequence, we search for

forbidden combinations of kth order using a modified Iterated Function System plots of chaos game. We demonstrate the efficacy of this method to analyze the available protein sequences of various viruses including SARS-CoV-2. We found that these ubiquitous forbidden orders are unique to each of the viruses we analyzed. This unique structure of the viruses may provide an insight into the genomic character of the species. Here we show the application of the method in the analysis of species evolution patterns, phylogenetic comparison, and virus mutations. This finding may have a broad significance for the analysis of coding sequences of species in general.

Keywords: Iterated function systems; Chaos Game; Forbidden order; COVID-19; Protein sequence.

Noise-induced calcium patterns in a biophysical model of astrocytic process

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The widespread consensus argues that there is undeniable bi-directional astrocytic regulation of neuronal activity at the cellular and network levels. Recent experimental studies have shown the high complexity and diversity of calcium dynamics in astrocytic process. The mechanisms of activity pattern formation are still the subject of intensive research. In this work, we explored a biophysical model of an extensional astrocytic process with organelle-free perisynaptic lamellae (leaflets). By varying the parameters of these leaflets, we changed the magnitude of the stochastic calcium entry through its plasma membrane from the extracellular space and further we revealed a nontrivial characteristics of calcium signal propagation along the process.

Keywords: Astrocytic process, Compartment, Calcium oscillation, Inositol 1,4,5 - trisphosphate, Diffusion, Astrocytic leaflet, Noise-induced pattern.

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The Turing Model and Discrete Limit Cycles with Eddy and Convection

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The Turing model and discrete limit cycles are considered in this paper, from the viewpoint of chaos functions. Firstly, the Turing model is explained as a reaction-diffusion system of chemical substances, and a three-dimensional (3-D) time-dependent solvable chaos map corresponding to the model, is derived on the basis of chaos function solutions. Secondly, a 2-D chaos map for the 2-D Turing model is proposed for simplicity, in order to present chaotic dynamics, and the 2-D map is shown to have symmetric bifurcation diagrams, a ring of cells and limit cycles with different patterns, depending on the system parameter. In particular, the limit cycles appear in pairs, and are discussed on left-handed and right-handed eddies, which generate convections, as nonlinear dynamics of non-equilibrium open systems.

Keywords: Turing model, Turing pattern, Chaos function, Bifurcation diagram, Symmetry, Fixed point, Limit cycle, Eddy, Convection, Non-equilibrium open system.

Bandgap Engineering in Graphene Using Quantum Chaos Approach

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Two-Dimensional (2D) materials often exhibit far more distinguished properties than their 3D counterparts and offer great potential to advance technologies. However, even graphene, the star of 2D materials, still face

several challenges, despite its high mobility and high thermal conductivity. One of such challenges is the lack of a bandgap for use in electronics, photonics or photocatalysis. Here, with the Quantum Chaos Approach, we propose methods for engineering and creating energy gaps in graphene. Several vacancy/ Boron configurations were considered in the study. We considered a zigzag graphene nanoparticle that contained 5,814 carbon atoms. First, we plotted the Level Spacing Distributions P(S) graphic diagram using MATLAB software for this nanoparticle, without adding any impurities. Then once again we plotted the P(S) chart, this time by adding boron impurities to the zigzag graphene nanofibers. By adding boron impurities to the zigzag graphene nanofibers, energy transitions and gaps have been created.

Keywords: Quantum Chaos, energy gaps, Level Spacing Distributions, zigzag graphene, bandgap, Engineering, Chaotic simulation.

Chaotic behaviour of vehicular traffic under dynamic reactive assignment and instantaneous network state information

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Vehicular traffic is still increasing in volume. Actually, the present Covid epidemic induces a perceptible disaffectation for public mass transportation. Consequently, the modal share of private car increases. Thus, it is crucial to manage efficiently traffic on a large scale. Typically, what is considered is a regional sized network, such as the Ile-de-France network in which the number of daily car trips exceeded 14.5 million in 2018. The new systems of information (floating vehicle data, internet/portable operators, crowd sourcing, and forthcoming vehicle-to-vehicle communication) provide instantaneous travel time and network state information. They provide a communication, both instantaneous and long-range, between travelers in the network. Such information can be used by public or private agencies to manage traffic. Indeed congestion depends strongly on traffic assignment, that is to say the choice of departure time and the path choice of travelers. The standard behavioral

assumption for departure time and path choice is that the traveler minimizes his travel cost (travel time + monetary cost). The travel cost expresses the network supply, which in turn depends on the traffic assignment. Those paths, which are attractive in terms of their cost, attract much demand and thus may see their attractiveness diminish. Strong nonlinearities in traffic dynamics result. In this contribution we consider reactive dynamic assignment, i.e. assignment in which travelers adapt their path choice "en route", as they receive real-time travel information from various agencies. The impact of travel information systems on assignment dynamics was analyzed in Han et al. 2011 and leads to possibly chaotic behavior. Chaotic behavior in path choice was also pointed out in Guo and Huang 2009. The choice of departure time need not be stable as shown in Iryo 2019. Current information systems provide the so-called "instantaneous travel times", which express the current state of the network. The reason is simple: such instantaneous travel times are easily produced by current sources of network information. On the other hand, effectively optimizing traffic flow and particularly traveler choice would require the use of actual predictive travel times. Actual predictive travel times are difficult to evaluate, they are also difficult to use because formally they have a semi-group structure. Liu et al 2018 consider the impact of various types of information on combined departure and path choice in a simple setting, showing potential chaotic behavior. When the path choice is determined based on instantaneous travel time information, strong nonlinear feedback and dis-utilities occur (Khoshyaran and Lebacque 2020). The object of the contribution is to analyze vehicular traffic as a complex and potentially chaotic system when the dynamic assignment process is governed by instantaneous travel time and traffic state information. The complexity of the system is increased by the fact that information providers may compete or at the very least not cooperate. The proportion of travelers benefiting from instantaneous network information constitutes a parameter of the study. Travelers who do not benefit of instantaneous information are assumed to rely on historical information. The study is based on a simplified model that still recaptures essential features of the system in terms of flow and traveler choices.

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Keywords: Nonlinear dynamical systems, infinite dimensional systems, complex system, fixed points, equilibria, stability, control, bathtub model, point queue model.

Modeling the Evolution of a cluster of gravitating bodies taking into account their absolutely inelastic collisions

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Numerical simulation of evolution of a cluster of a finite number of gravitating bodies interacting only by their intrinsic gravity has been

carried out. The goal of the study was to reveal the main characteristic phases of the spatial distribution of material bodies constituting the cluster. In solving the problem, the possibility of interbody collisions was taken into account, the collisions being assumed to be absolutely inelastic. Forces external to the body cluster under consideration were ignored. Among all the internal force factors acting within the cluster, only the gravitational interaction was taken into account. The total mass of all the gravitating bodies of the cluster was assumed to remain constant during the entire evolution. The Cauchy problem with natural initial conditions was considered. To check the process of solution, the so-called rotation curve was used which represents the current radial distribution of orbital velocities of the cluster bodies. The numerical analysis showed time variations of the model cluster rotation curve and, particularly, the fact that the rotation curve horizontal section is only a short moment in evolution of the gravitating bodies cluster. The results obtained within the scope of classical mechanics show that it is possible to represent all the rotation curve variations for the observed galaxies without appealing to the hypothesis of non-observable gravitating "dark matter".

Keywords: Galaxy rotation curve, N-body problem, Evolution of gravitating masses, Evolution number, "dark matter".

Evolution of bifurcation boundary in Painleve-2 equation

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Solutions of the perturbed Painleve-2 equation are typical for describing a dynamic bifurcation of soft loss of stability. The bifurcation boundary separates solutions of different types before bifurcation and before loss of stability. This border has a spiral structure. The equations of modulation

of the bifurcation boundary depending on the perturbation are obtained. Both analytical and numerical results are given.

Keywords: bifurcation, perturbation theory, integrability, Painleve equation.

Designing efficient echo state reservoirs for chaotic time series prediction

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Reservoir models, such as echo state networks, have shown unique efficiency in chaotic time series classification and prediction. In these models, the reservoir is represented as a set of nodes connected into an oriented graph. In classic echo state networks, the graph serving as a computational reservoir is randomized, but intuitively, certain types of non-random graphs could offer better performance. In this paper, we use genetic algorithms and local unsupervised edge rewiring to construct quasi-random graphs that can serve as effective echo state reservoirs. We use these echo networks to predict chaotic time series, and apply classic network and computational topology measures, such as Ricci curvature, to describe the properties of computationally effective reservoirs. Taken together, our work offers practical steps towards the design of efficient reservoir models.

Keywords: Echo state networks, chaotic time series prediction.

Nonlinear Model for Strongly Localized Wrinkling Modes of Mono- and Few-Layer Graphene Sheets in or on a Strained Matrix

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Exceptional mechanical properties of graphene have made it very attractive for the construction of nano-mechanical devices and as a reinforcing inclusion in polymer nanocomposites. The nonlinear model describes the characteristic features of the wrinkling modes, localized in the plane of mono- and few-layer graphene sheets embedded in or placed on a compliant compressively strained polymer matrix. The proposed model, based on nonlinear elasticity of the graphene sheets, shows that beyond the critical compressive surface stress the spatial localization of the wrinkling mode occurs with soliton-like envelope with localization length, substantially decreasing with the overcritical compressive surface stress. Two types of graphene wrinkling modes with different symmetry are described.

The work was supported by the Russian Foundation for Basic Research, Grant No. 18-29-19135.

Keywords: Nonlinear elasticity, Mono- and few-layer graphene sheets, Polymer matrix, Wrinkling modes, Strong localization, Wrinkling mode symmetry.

Nonlinear Localization in Lattices.

Special Session Chair:

JFR Archilla (Seville, Spain), or

Yuriy A. Kosevich (Moscow, Russia)

This Special Session will be devoted to the presentations on different aspects of nonlinear mode localization in different types of lattices and systems, including low-dimensional and nanoscale. The Special Session with the same title was organized by Prof. JFK Archilla from the University of Seville, Spain, during the online CHAOS-2020 International Conference.

Optimization approaches when calculating the “massif - innovative fastening parameters” spatial system

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Efficiency and safety are the main components of the requirements to the solid fossil fuels production industry. Currently, the combined use of

alternative energy sources and innovative resource-saving technologies is relevant. Optimization approaches ensuring the stability of the “massif - innovative fastening parameters” spatial system elements are studied. The principles and methods of resource-saving increase in stability on the basis of minimizing the intensity of the rock pressure manifestation by adjustable operating modes of innovative fastening systems have been scientifically substantiated. The resulting developments make it possible to achieve the technological processes intensification during the minerals extraction and to develop a method for calculating a function that describes the rational deformation-strength characteristic depending on mining-and-geological conditions.

Keywords: Optimization approaches, Stability, Resource saving, Minimization, Spatial system.

Nonideality of a parametric system as a trigger of chaos

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There are several characteristics common to all resonance vibrations occurring in systems with a circumferential coordinate (rings, cylindrical and spherical shells, etc.). The specifics of resonance phenomena make it possible mathematically to reduce the problem to the study of vibrations in distributed systems and examine an analogous problem with fewer dimensions. In the current theory of dynamical systems the coupling of can be ignored without altering the qualitative pattern of the phenomenon only when the coupling does not produce a bifurcative change in the stability of the process as a whole. After analyzing averaged equations for systems with a circumferential coordinate in cases where different types of resonance loads were applied, we were able to classify the resonances that occur and determine that chaos is possible only with forced resonance excited by loading without interaction with source of energy or it is possible in the systems under nonideal excitation.

Keywords: Systems with a circumferential coordinate, Interaction with source of energy, Steady state regimes, Chaos.

Synergetic control of the separation of the upper stage and the carrier aircraft with non-simultaneous breaking the connections

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A synergetic approach to the synthesis of a control system for a nonlinear dynamic object is proposed on the basis of the well-known method of analytical design of aggregated regulators - ADAR. An applied problem of synthesis of control algorithms for the separation of the upper stage and the carrier aircraft when the nose link is broken and the upper stage rotates at the main nodes is considered. The upper stage is located on the upper surface of the carrier aircraft and is connected to it by three nodes, the bow and two main ones. The carrier aircraft with the upper stage makes a flight to a given altitude, at which they are separated (air start of the upper stage). At the moment of separation, the nasal connection is broken. When the elevator is deflected and under the influence of the incoming flow, the upper stage rotates by a predetermined angle around the axis passing through the main attachment points. This angle is set so that the aft parts of the upper stage and the carrier aircraft do not collide. The obtained control algorithm provides stabilization of the upper stage rotation angle and compensation of external piecewise-constant disturbances acting on the system. The effectiveness of the synergetic approach is confirmed by the asymptotic stability of the closed-loop system "object of control - regulator", as well as its invariance to the influence of disturbances in the external environment, which is clearly demonstrated in numerical studies of the synthesized system.

Keywords: synergetics, synthesis of control algorithms, piecewise constant disturbances, aircraft carrier, upper stage.

Synergistic synthesis of aircraft braking control on the landing strip in difficult weather conditions

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One of the applied problems of automatic control of an aircraft is the synthesis of control systems for aircraft landing (braking on the ground) under adverse weather conditions. At the same time, the landing strip is polluted by atmospheric precipitation and its condition is not known in advance. It can be covered with slush, water, wet or compacted snow. The thickness of the coating can vary with the length of the strip. Uneven precipitation coverage can create different operating conditions for the left and right brake wheels. Thus, the problem arises of adapting the actuators of the aircraft braking system to the corresponding mode of movement along the lane. The key problem is the synthesis of control laws that provide invariance to environmental influences and the optimal ratio of deceleration and slip modes when the aircraft moves along the runway. We present the method of synergistic synthesis or analytical design of interrelated laws of aircraft braking control with runway surface state real-time identification. This laws account aircraft dynamical properties as nonlinear mechanical plant in difficult weather conditions.

Keywords: synergetic synthesis of control laws, aircraft braking on the runway, friction coefficient real-time identification.

Filter for Ultra-Wide-Band chaotic signals of the microwave band

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The application of chaotic signals in wireless communication systems is considered. A method for filtering ultra-wideband chaotic radio pulses in the receiver is proposed. The filter in the receiver is a passive frequency selective circuit that generates a chaotic signal in the chaos generator. Due to the consistency of the characteristics of the filter and the chaotic signal, an improvement in the criterion of the probability of a bit error is achieved.

Keywords: Chaotic oscillators, chaotic ultrawideband signals, chaotic radio-pulses, wireless communications, chaotic signal reception.

Motivation and professional satisfaction of medical and nursing staff of Primary Health Care structures (Urban and Regional Health Centers) of the Prefecture of Heraklion, under the responsibility of the Ministry of Health, 7th Health district of Crete

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in this article we study the motivation and professional satisfaction of employees in their work environment which is one of the most critical issues of modern administrations as it is inextricably linked to their efficiency and effectiveness and therefore to the viability of companies / organizations. These two determinants are critical on the one hand to the emotional and psychological state of the employees and on the other hand to the proper functioning of a company or an organization. The purpose of this master's thesis was to study the motivating factors that affect the job satisfaction of doctors and nurses of Primary Health Care.

The project begins by listing the basic theoretical approaches to job satisfaction and motivation and the main factors influencing these variables. It turns out that supervision, camaraderie and the nature of the work play a main role in contrast to the lower pay and operating conditions. In the scale of motivation, the highest average is presented by the characteristics of work and colleagues, while the lowest percentage is displayed by the scale of successes. In between, the scale of colleagues and earnings is recorded. Important elements are that non-monetary incentives such as job characteristics, supervision, camaraderie are main factors for both variables job satisfaction and motivation.

Application of Fractal Analysis Elements in the Theory of Finance on the Example of the Russian Stock Market and COVID-19

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In this paper, we apply the fractal market hypothesis to price series using R/S analysis. The fractal market hypothesis was introduced by Peters in the early nineties [1]. It represents the evolution of the effective market hypothesis. We give a retrospective estimation of correlations between the Hurst exponent dynamics and asset prices for main stock Russian indices. The cycle length or number of points that provide the best memory effect was found to be 300 [2]. In this work, we used the data of the asset prices of the RTS and MICEX indices for more than twenty years from February 24, 1999 to April 17, 2020 including the financial crises which took place in 2008 and dramatical events of the beginning of 2020 originated from the COVID-19 pandemic. The daily closing prices were used as points. Using the sliding window method, we calculated the values of the Hurst exponent for each of the days over the specified period. We analyzed the daily closing asset prices for the MICEX and RTS indices using the rescaled range. The real prices values are compared with the dynamics of the Hurst exponent and some predictive features of the latter have been discovered. In most significant cases including the 2020 crisis we can see tendency of the Hurst exponent to approach 0.5 value before a collapse occurs. It indicates that there are changes in the ratio between long-term investors and short-term speculators. Fundamental analysis of the stock assets works no more and prices begin following unpredictable fluctuations. Thus, the fractal market hypothesis in combination with R/S analysis makes it possible to explain and predict possible significant changes in market behavior in the near future.

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Keywords: Hurst Exponent; COVID-19; Financial Crisis; Fractal Market Hypothesis; R/S Analysis.

Cloud electrification as an energy ignition source for hydrogen lift-gas airships

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In 2020, the authors of this paper gave a presentation at the 13th virtual chaos 2020 conference Florence-Italy on how Peek's formula [1] may be used to estimate the visual inception voltage stress point on natural and artificial structures [2] and microwave oven plasma processing of nanomaterials [3]. In the former paper, the dirigible airship was one of the artificial structures examined. In the follow-up question and answer section, the main question was 'if not lightning, how does Saint Elmo fire become a lethal threat to hydrogen lift-gas airships? The answer to this question is complex, involving a detailed knowledge of airship construction, prevailing metrological conditions at the time and if the airship's captain was 'Extraordinary Good', or 'Lucky'. Answering this question is not a purely academic exercise, but has relevance today's hydrogen economy designed to reduce world energy consumption and greenhouse gas emissions whilst removing the 'Hindenburg Syndrome' from public perception. Over the past 25 years, Saint Elmo's fire (sometimes-called brush discharge, corona, or partial discharge) has been proposed as a potential ignition source for hydrogen lit-gas airship disasters in-flight disasters, rather than lightning strikes as portrayed by the popular press. This paper reviews $H_2 + O_2 = H_2O$ combustion chemistry, the role of heterogeneous graupel chemistry within electrification of Cumulonimbus, and how the empirical mathematical construct Peek's Law is used to identify the minimum electrical field stress required for the generation of discharge at sharp protrusions. Using this electrochemical knowledge, disaster associated with nearby cloud electrification, or violent storms systems, is correlated and reviewed with firsthand accounts (from survivors), radio messages prior to the airships disaster, ground eyewitness accounts, along with supporting evidence from the structural design of the airship. The hydrogen lift-gas airships reviewed here are four dirigibles (LZ-4, SL-9, Dixmude and Hindenburg) and one non-rigid airship (NS.11). As a comparative control, a review of

the worst airship disaster, that of the helium lift-gas flying aircraft carrier, USS Akron, which led to the loss of 73 lives) is included.

References used are from the time of the disasters (popular press, board of inquiry and aeronautical journals) to current aeronautical, metrological, physical chemistry and electrical engineering journals. The chronological ordering of these documents reveals how the complex processes (commercial, political and physical) involved in the disasters were not readily identified, but revealed overtime.

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Keywords: Saint Elmo's fire, plasmoid, dirigible, non-rigid airship, lightning.

'Dubro' Resophonic Guitar: Glissando Gestures

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Whether in the Hawaiian [1], Bluegrass [2], Rock'n'roll [3], film sound track [4] or animated cartoon [5] genre, the glissando sound made on a slide-guitar is one the most instantly recognizable sounds in western music. This paper reports on the complex acoustical and perceptual glissando of the opening seconds of Warner Brothers 'Looney Tunes' (LT) ascending glissando and its counterpart (descending glissando, both played on a 'Dubro' resophonic guitar [6, 7] as a function ascending (LT) and

descending glissando (PT) for both steel and glass bottleneck. With the resophonic guitar tuned to open G (D-G-D-G-B-D), ten recorded acoustic tracks are analyzed using the toolbox within Audacity software (time base, standard autocorrelation, spectrogram, or waterfall plot (time, frequency and acoustic energy) and noise reduction). The tracks are transcribed for tempo, consonant, dissonant, string squeaks throughout the psychoacoustic warm, bright and brilliance / presence regions.

The transcription identifies three distinct acoustic signatures.

1. String squeaks due to the guitarist's gesture are identified. These are associated with applied bottleneck pressure to the strings and subsequent pull-off. Within these time-stamped events are the cognitive tactile and auditory guitarist feedback responses.

2. The dynamic slide movement divides the string scale length into two coupled longitudinal vibrating segments, each producing a coherent continuous mirrored exponential varying pitch that extends up to 15 kHz within the guitar's brilliance region.

3. Incoherent or hiss-like noise is found within the lower (0 to 0.5 kHz) warm region. This incoherent noise is linked to a slip-stick friction process between the slide and string. Slide material and slide direction varies the intensity of the noise that has a Voss-Clarke $1/f$ -like response [8] with a Brownian ~ -7 dB/10 Hz roll-off.

The study provides an insight into the complex psychoacoustic tonal quality of a resophonic guitar by mapping the incoherent noise within the warm, bright and brilliance regions of the guitar that adds to the listener's perceptual experience. For example, descending glissando invokes a psycho physiological response (relaxation or even sadness) as found in 'Blue Hawaii' and 'Paris, Texas', whereas an ascending glissando is well known to invoke joy and laughter.

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Keywords: Dubro, resophonic guitar, bottleneck, glass, steel, glissando, noise.

Theoretical Foundations of Fractal Electrotechnic. Fractal Elements and its Properties

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The article is devoted to the foundations of the theory of fractal electric circuits, which generalizes the traditional theory of circuits to the case of the relationship between currents and voltages of elements by derivatives of arbitrary order, including fractional ones. The dependence of the active and reactive components of the complex of the impedance of the fractal element on the degree of the derivative is investigated. The analysis of properties of ideal fractal elements is carried out. Based on this analysis, it was concluded that the traditional resistor, inductor and capacitor are special cases of a fractal element in the basis of degrees of derivatives (1, 0, -1). It is proved that the units of measurement of resistance, capacitance and inductance are a special case of the general unit of measurement of fractal elements parameter. This general unit can be used for any values of the degree of the derivative. It is shown that, unlike traditional ideal elements, a fractal element can have unusual properties, such as the dependence of active resistance on frequency or the presence of negative active resistance. A family of frequency characteristics of a fractal element is constructed depending on the degree of the derivative. The results obtained in the article can be the basis for studying the properties of fractal electric circuits in various **operating modes**.

Keywords: Fractal derivatives, Fractal elements, Fractal electric circuits, Charge conservation, Frequency characteristics.

Bifurcation theory of dynamical chaos in Hamiltonian and conservative systems

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It will be considered a new bifurcation approach to the analysis of solutions of perturbed Hamiltonian and conservative systems, which implies the construction of an approximating extended two-parameter dissipative system whose stable solutions (attractors) are arbitrarily exact approximations to solutions of the original conservative system. It will be shown on the basis of numerical experiments for several Hamiltonian and conservative systems such as conservative Croquette equation, Yang-Mills-Higgs and Mathieu-Magnitskii systems that, in all these systems, transition to chaos takes place not through the destruction of two-dimensional tori of the unperturbed system in accordance with KAM (Kolmogorov-Arnold-Moser) theory, but, conversely, through the generation of complicated two-dimensional tori around cycles of the extended dissipative system and through an infinite cascades of bifurcations of the generation of new cycles and singular trajectories in accordance with the universal bifurcation FShM (Feigenbaum-Sharkovskii-Magnitskii) theory.

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Keywords: Hamiltonian and conservative systems, dynamical chaos, FShM-theory.

The optimum health expenditure allocation in Greece

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The main part of the Healthy Life Years Lost Methodology (HLYL) is applied by developing and extending a life table important to estimate the healthy life years lost along with the fitting to the health expenditure in the case of Greece in 2009. The application results are quite promising and important to support decision makers and health agencies with a powerful tool to improve the health expenditure allocation and the future predictions.

Business Strategy and COVID-19: Evidence from Crete

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In this article, we examine the strategy followed by the businesses from Crete during the COVID-19 pandemic. The aim is to examine how the firms are financially protected. Also we investigate and examine the state measures which they consider for the management of COVID-19 crisis. The businesses seem to re-examine their strategy and their budget, cut on expenses and reassess their estimated revenue. More specific, concerning the measures that the state must take amidst this crisis, the businesses should impose measures for their financial support.

Similarity measures for the classification of dynamical changes in time series of different origin

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Dynamical features of real processes in most cases are assessed by relatively short time series. On the other hand, usually, most of standard approaches in modern data analysis require long time series of high quality. These requirements usually is difficult to be fulfilled for real world measurement time series. This is why in last decade we observe increase of research interest in the combination of modern data analysis views with the well known statistical concepts of similarity measures. One of such example, is the convenient for short time series approach, which is based on Mahalanobis distance calculation combined with surrogate time series testing. This metric is important in classification problems because incorporates the information about spatial distribution of the points. The similarity or dissimilarity is assessed by a number that measure how distinct data sets are. The greater the number, more distinct the data sets are. Not the less similarity measure is the Kullback-Leibler divergence. This is a measure based on the relative entropy of two probability density functions build for considered processes. In this research we have compared Mahalanobis distance metric and Kullback-Leibler divergence for the identification of two dynamical processes. In order to test ability of these two approaches to differentiate changes occurred in complex processes we used model and measured time series of different origin. First we used Mahalanobis distance metric and Kullback-Leibler divergence for short datasets generated by several low dimensional attractors like Lorenz and Henon with the added white noises. We found that increase of noise intensity leads to the increase in measured metric values showing increase difference between considered time series (processes). We concluded that even slight changes in the original dynamics, caused by white noise, can be recognized by used separation measure procedures. Next, we proceeded to the analysis of the measured time series. Namely we used internet, physiological and economic data sets. Exactly, we analyzed Border Gateway Protocol (BGP) recordings time series from 4 autonomous system, arterial systolic and diastolic blood pressure time series of healthy persons and patients with arterial hypertension collected at Institute of Clinical Cardiology, Tbilisi, Georgia, as well as components of Index of Economic Freedom (IEF) and exchange rate time series and of three southern Caucasian countries obtained from corresponding international data bases. Earlier analysis of these data sets

indicated quantifiable nonlinear signature in their dynamical properties. Thus, we aimed to learn whether these processes, or what is the same considered time series, are similar or dissimilar by measures provided by Mahalanobis distance and Kullback-Leibler divergence metrics. Based on results of analysis we conclude that used data analysis methods, when they are combined with the surrogate testing, enables recognize slight changes occurred in the dynamics of considered short model as well as real time series.

Design of Ergodic Maps with Prescribed Invariant Densities and Multimodal Power Spectra

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The inverse Frobenius-Perron problem of designing an ergodic map with prescribed statistical characteristics is frequently encountered in dynamical system modelling and simulation. Solutions for this problem have led to a model for neurotransmission in the olfactory system, more efficient random number generators and more durable clogged bits for rock drills. Growing demand for more realistic system models is creating interest in solving a joint formulation of this problem, where both the invariant state density function and the power spectrum of the map are prescribed. Whereas a numerical approach to solving this joint problem has been proposed, a solution with guaranteed converge and that accommodates the selection of an arbitrary power spectrum remains elusive. In this paper, we report on recent progress towards an analytic solution for the joint problem. A previous solution strategy, which draws an analogy between the joint problem and the problem of designing an

ergodic Markov chain with equiprobable states and a state transition matrix with a prescribed real-valued eigenspectrum, is used as a starting point. In this strategy, a piecewise defined map with linear branch functions and which satisfies the Markov property over a partition of its domain is designed such that the matrix representation of its Frobenius-Perron operator is equal to the state transition matrix of the Markov chain. We address this strategy's inability to realise smooth invariant density functions and spectral modes with arbitrary centre frequencies, as well as its realisation of undesired spectral modes. Specifically, we present a novel Markov state disaggregation algorithm for generating a state transition matrix with a prescribed complex-valued eigenspectrum, as opposed to the real-valued eigenspectra associated with existing disaggregation algorithms. It is shown that this improvement accommodates the selection of a multimodal power spectrum with modes having arbitrary centre frequencies, as well as selection of the mode bandwidths. A method for designing a piecewise non-linear hat map, such that the matrix approximation of its Frobenius-Perron operator equals the state transition matrix, is proposed. It is shown that the symmetry and non-linearity of the resulting map's branches eliminate undesired spectral modes and accommodate the selection of a smooth invariant density function. The solution is demonstrated by generating ergodic maps with invariant density functions from several well-known parametric probability distributions and power spectra having more than two modes. It is anticipated that the proposed solution for the joint inverse Frobenius-Perron problem will empower researchers to develop more realistic models of dynamical systems.

Keywords: Inverse Frobenius-Perron problem, ergodic map, invariant density, power spectrum, Markov state disaggregation.

Stability of the Similar Viscoelastic Telegraph Problem Governed by Lamé Operator

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In this talk we will investigate the stability of similar viscoelastic telegraph problem governed by Lamé operator.

We denote by Ω an open subset of \mathbb{R}^n with regular boundary Γ . Let Q the cylinder $\mathbb{R}_x^n \times \mathbb{R}_t$ with $Q = \Omega \times]0, T[$; T fini, Σ boundary of Q , $u_0(x)$ and $u_1(x)$ are functions. We look for the stabilisation of a function $u = u(x, t)$, $x \in \Omega$, $t \in]0, T[$, solution of the problem (P).

$$(P) \begin{cases} \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u - Lu + \int_0^t g(t-s)\Delta u(s)ds = 0 & \text{in } \Omega \times]0, T[\\ u(x, t) = 0 & \text{on } \Sigma \\ u(x, 0) = u_0(x), \quad \frac{\partial u(x, t)}{\partial t} \Big|_{t=0} = u_1(x) & x \in \Omega \end{cases}$$

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Keywords: Energy, Lamé, Lyapunov, Stability, Telegraph, Viscoelastic.

Attractor Coexistence in Extended Lorenz Systems Revealed Through Bifurcation Analysis

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Phase space of a dynamical system can sometimes be shared by two or more coexisting attractors. Coexisting attractors may arise from many different sources such as hidden basins of attraction or peculiarly organized bifurcation structures. For Lorenz and Lorenz-like systems, one can exploit mismatches in the bifurcation structures for local and global bifurcations to find such coexisting attractors. In this study, we focus on the so-called “physically-extended” Lorenz systems, which are high-dimensional extensions of the Lorenz system obtained from incorporating additional physical ingredients such as rotation and density-affecting scalar in the governing equations for Rayleigh-Bénard convection problem. These newly considered physical ingredients can influence the bifurcation structures as well as the attractor coexistence characteristics of the system. Inside the regions of potential attractor coexistence, coexistence of periodic and point attractors and that of two different periodic orbits are found, in addition to the well-known coexistence of chaos and stability prior to the Hopf bifurcations of the nontrivial fixed points. The coexisting attractors identified in this way are visualized using solution trajectories and three-dimensional renderings of basin boundaries.

Keywords: Lorenz-Stenflo system, high-dimensional Lorenz system, coexisting attractors, basin of attraction, Hopf bifurcation.

Modelling Synchronization of Large-Scale Modes in Fluid Systems

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In electrically neutral fluids or in plasma flows the presence of nonlinear interactions can lead to the development of turbulence. In general, turbulence is characterized by energetic couplings between different scales of a flow. However, in the context of turbulence driven transport, such as the case of magnetically confined fusion plasmas or the diffusion of cosmic rays, typical flow structures are identified by dominant modes and the global turbulent state is approximated by a superposition of linear contributions (waves in general). These theoretical studies consider the amplitudes of the fluctuating quantities, but disregard the dynamics of the phases by using the so-called random-phase approximation (RPA) for which the existence of a Chirikov-like criterion for the onset of wave stochasticity is assumed. In this approximation one assumes that the dynamical amplitudes have a slow variation compared to the rapid change of the phases, which are considered to be distributed uniformly over a 2π interval. It has been observed that the phase dynamic shows significant departure from the well-known RPA assumptions, with phases locking occasionally (but not in the dissipative high- k range). In the well-known Kuramoto non-linear system with two-body interactions of limit cycle oscillators, it was shown that these types of systems are prone to locking if the coupling strength between the two-bodies has passed a threshold. In non-linear turbulent flow however, three-body interactions between the phases of the various modes is of importance. We will consider examples of synchronization in different fluid system such as Burgers and Navier-Stokes turbulence and in more advanced models such as those for Edge Localized Modes (ELMs) in tokamaks which remain a critical issue for plasma stability and the lifetime of fusion reactors such as ITER. The dynamic of the three-body interactions between the phases in the non-linear Burgers' turbulence differs from the simplified picture of Kuramoto, and the phases lock intermittently and only in the low to mid- k range.

Keywords: Kuramoto, oscillators, synchronization, turbulence, ELM.

Correlation is not causality: a conceptual and mathematical framework

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Causal relations are a crucial aspect of the human understanding of the world. On the other hand, most statistical and machine learning tools are completely blind to the distinction between correlation and causality. This lack of discrimination capability can be catastrophic for control, particularly of complex and chaotic systems. In this contribution, a conceptual framework is provided to distinguish between correlation and causality. A new definition of causality for the sciences is proposed. How this can be converted into mathematical criteria is also covered. The extraction of causal relation directly from the data, the field of so called observational causality detection, is introduced as well.

Keywords: Observational Causality Detection, Time Series, Nonlinear Interactions.

Methods and algorithms for the analysis of chaotic and fractal structures in dynamic processes of formation of classes of prime numbers in the generalized Artin's hypothesis

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It is proved that any natural number other than one is the basis for the classification of the set of primes by the value of the index. It was found that the processes of class formation make it possible to estimate the values of the generalized Artin's constants. The chaotic and fractal structure of the dynamic processes of the formation of classes of prime numbers as a function of the base value and class index is investigated. Algorithms and methods for estimating the generalized Artin's constants were developed and the convergence of the estimates for the constants in probability to the limiting values was proved.

Keywords: Generalized Artin classes, Artin constants, Class probabilities, Stability of estimates of the Artin constants, Convergence in probability.

Nonlinear Feedback Controller For Adaptive Generalized Hybrid Projective Synchronisation between Two Identical Chaotic Systems

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Synchronisation of chaotic systems is an important research problem in chaos theory. In this research work, a novel generalised hybrid synchronization of identical uncertain chaotic systems is investigated, where the master system is synchronized by the sum of hybrid state variables for the slave system. According to the Lyapunov stability theorem, a new adaptive nonlinear controller for the synchronization is designed, and some parameter update laws for estimating the unknown parameters of these systems are also gained. The synchronization between two identical Zeraoulia systems and two identical Vaidynathan systems are studied to show the effectiveness of the proposed method.

Keywords: Adaptive Control, Chaotic systems, Hybrid Synchronisation, Chaos.

Application of Recurrence Quantification Analysis (RQA) on astronomical data

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Behind the acquisition of astronomical data, there is a lot of effort and use of many resources. The RQA is a method of non-linear data analysis and therefore is also in strong relationship with the chaotic phenomena.

In this work, novel techniques using RQA are presented, mainly on observational data of astronomical origin. This kind of data have special

characteristics as uneven data distribution, high noise content, short length. This work mainly based on [1-3], shows how RQA can be used in order to get the most information from this kind of data.

Keywords: RQA, astronomy

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Supergranulation and the Influence of Magnetic Field

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I study the complexity of supergranular cells using intensity patterns from Kodaikanal solar observatory. The turbulent aspect of the solar supergranulation can be studied by examining the interrelationships amongst the parameters characterizing supergranular cells namely size, horizontal flow field, lifetime, Area, Perimeter and its Fractal dimension. The Data consist of visually identified supergranular cells, from which a fractal dimension 'D' for supergranulation is obtained according to the relation $P \propto A^{D/2}$ where 'A' is the area and 'P' is the perimeter of the supergranular cells. I find a fractal dimension close to about 1.3 which is consistent with that for isobars and suggests a possible turbulent origin. Fractal dimension of the supergranular cells also shows a latitudinal dependence. The findings are supportive of Kolmogorov's theory of turbulence. Since supergranular cells are essentially a manifestation of convective phenomena, they can shed light on the physical conditions in the convection zone of the Sun. Moreover, supergranules play a key role

in the transport and dispersal of magnetic fields as it is an important step in our quest to understand the solar cycle.

Joint effects of chaos and noise in Josephson junctions

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We report about various chaotic phenomena in Josephson junctions. The investigations are performed both theoretically and experimentally. In modeling, we consider complex dynamics with interplay between chaotic regimes and thermal noise. We demonstrate that chaos can significantly increase the spectral linewidth, which is important for Josephson junctions application as THz oscillators [1]. HTSC long Josephson junctions demonstrate diverse nonlinear dynamics. The current-voltage characteristics of YBCO junctions have been measured at various magnetic fields and temperatures in the presence of external ac driving. The Shapiro steps with a number of fractional substeps have been observed. Their appearance is explained by quasi-chaotic dynamics of the weak Josephson soliton chain at low magnetic fields, which is confirmed by the numerical solution of the sine-Gordon equation. We demonstrated that fractional steps disappear at larger magnetic fields, making the vortex chain stronger. In Al Josephson junctions at mK temperatures [3], subjected to ac driving, complex quasi-chaotic dynamics may develop in the crossover between classical and quantum regimes. The work was supported by the Russian Science Foundation (project 19-79-10170).

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Keywords: Josephson junction, chaotic regimes, noise.

Environmentally induced chaos and synchronization of the neuronal activity

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Nerve system is a highly complicated multielement system composed of neuronal cells, glial cells, perineuronal nets, extracellular matrix and other secreted molecules placed in an interstitial fluid. Obviously, to provide well-being of all structural elements in vivo and to support successful cellular cultivation in vitro, an appropriate culture medium with optimal parameters, in particular, with optimal interstitial viscosity, is needed. Simple phenomenological model for the neuronal network with two types of the coupling between the cells was recently introduced in [1]. The phenomenon of amplitude death for the small-scale networks was revealed, and analytical estimation for the threshold network scale providing the neuronal activity was obtained. In this study, the role of common environment in emergence of chaos and synchronization of neuronal activity has been examined. Different bifurcation scenarios for transition to irregular firing activity have been shown. Environmentally induced multistability with co-existent regular and chaotic regimes of neuronal activity has been revealed. Role of interstitial viscosity in environmentally induced complete synchronization of neural cells has been discussed.

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Keywords: Neural network activity simulation, Chaos, Synchronization.

Using Machine Learning Techniques for asserting Cellular Damage Induced by Ionizing Radiation

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Ionizing radiation (IR) is considered a genuine type of exogenous environmental stress to any organism, especially at medium to higher doses. IR introduces a complex response on living organisms and tissues challenging the homeostatic mechanisms. The outcome of IR depends to a multitude of physical and biological factors and ranges from cell death to many types of DNA damage. This study aims to employ Machine Learning (ML) techniques to ascertain the impact of particle ionizing radiation to cell survival and DNA damage. Our goal is to create a ML-based model that predicts the alpha and beta coefficients of the well-known and established Linear Quadratic (LQ) model and key metrics of DNA damage induction. The aim was the development of a computational prediction framework that is able to assess key radiobiophysical quantities taking into account the intrinsic complexities, interactions and stochastic effects, and on the other hand interpret bio-physically the outcomes of the model. Current empirical models do not always take into account such interactions, thus our work will provide a useful prediction model and interpretation tool. The data that the study is based on, is a publicly available dataset of irradiation studies. All types of complex DNA damage (All Clusters) is calculated using the fast Monte Carlo simulation code (MCDS), as well as, the number of DNA double strand breaks (DSBs) which are widely accepted to be closely related to cell survival and the detrimental biological effects of IR

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Accuracy Improvement of the Highest Lyapunov Exponent Estimation

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The highest Lyapunov exponent characterizes the degree of exponential divergence of close trajectories. The presence of a positive Lyapunov exponent in the system indicates a rapid divergence over time of any two close trajectories and sensitivity to the values of the initial conditions.

Therefore, the determination of the Lyapunov exponent makes it possible to identify a dynamical system as a system with chaotic dynamics in it. When studying the output signals of dynamic systems, it is often necessary to quantify the degree of randomness of the output signal when equations of the system are unknown. In this paper, the possibilities of accuracy improvement of the numerical algorithms of Benettin and Wolf for estimating the highest Lyapunov exponents of an attractor of a dissipative dynamical system are shown. Under these procedures a system itself can be specified both analytically (by a system of differential equations) and only by output signal.

Keywords: Highest Lyapunov exponent, Algorithms of Benettin, Algorithms of Wolf, Initial conditions, Chaotic modeling.

A 2D space-time discretization of a nonlinear peridynamic model

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Peridynamics is a nonlocal theory for dynamic fracture analysis consisting in a second order in time partial integro-differential equation. We consider a nonlinear model of peridynamics in a two-dimensional spatial domain. We implement a spectral method for the space discretization based on the Fourier expansion of the solution while we consider the Newmark- β method for the time marching. The approach takes advantages from the convolutional form of the peridynamic operator and from the use of the discrete Fourier transform. We show a convergence result for the fully discrete approximation and study the stability of the method applied to the linear peridynamic model. Finally, we perform several numerical tests and comparisons to validate our results and provide simulations implementing a volume penalization technique to avoid the limitation of periodic boundary conditions due to the spectral approach.

Keywords: Nonlinear peridynamics, Spectral methods, Newmark- β method, Nonlocal models.

Assessing Causality with Conditional Recurrence Plots

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In many fields of the natural sciences, from biology to physics, information tools are acquiring more and more importance. For the analysis of information transfer between time series in particular, the use of the transfer entropy is spreading. A typical application is synchronization experiments, which involve coupled quantities, a "target" and a "source", with quasi-periodic behaviours. On the other hand, in complex systems very rarely a couple of quantities can be really considered fully isolated and immune from other influences. It is therefore important to consider not only the relative influence of their past, but also the possible influence of additional factors. In order to tackle this problem, an advanced application of the recurrence plots, called Conditional Recurrence plots, has been developed. The innovative technique is corroborated by the application of the conditional transfer entropy. Preliminary results from experimental data of sawteeth pacing with radio frequency and ELMs pacing with Pellets in thermonuclear plasmas are very encouraging. Being quasi periodic, sawteeth occurs naturally and, especially in H-mode plasmas, the effectiveness of the pacing with RFF can be difficult to establish. ELMs also can occur naturally, without being triggered by an injected pellet. The proposed data analysis procedure is aimed at better isolating the effects of the external perturbations, like natural sawteeth or ELMs, providing both a more accurate quantification of the pacing efficiency and a deeper insight of the physical processes involved, thanks to a better understanding of the relevant causal relations.

Keywords: Nonlinear dynamic, Plasma Physics, Recurrence Plots, Conditional Recurrence Plots, Conditional Transfer Entropy, Nuclear Fusion, Synchronization.

Goodness-of-fit for Second Order Power Laws

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There is a core problem in model-informed Earthquake monitoring and early-warning systems for financial crises, both of which rely on power law expansions. These models are valid only for times near the phase transition. This makes it difficult to assess model fit because by measurements of Earthquake activity, financial volatility, and extremum of geophysical processes the present is certainly anomalous. There are two paths around this difficulty. One is using a third order (Landau) expansion with 9 free parameters that models second-order dynamics in frequency and lacunarity near the phase transition. In order for higher-order power law expansions to have relevance, there remains an important gap in the literature, and they remain experimental. The practical alternative fits second-order expansions and develops criteria for assessing model nonfit. We review these criteria, and propose a more intuitive alternative. Our alternative is compared in time series simulations and found to have similar efficacy without an assumption of universality. Our proposal is extended to a framework for model nonfit relevant to higher-order expansions.

Keywords: Chaos, Critical Phenomena, Statistical Dynamics.

Modelling the influence of RT and BBC on cognitive attitudes and psychophysiological indicators of individuals

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The information influence in the modern globalizing world is a serious challenge to the security of any state. This article presents the results of an experimental study of the way the modern media affect the cognitive attitudes of individuals on the example of two leading international TV channels - RT and BBC. In order to conduct this study our team developed an experimental plan for the psychophysiological recording of deformation

of cognitive attitudes under the external informational influence. The study was conducted at the Department of Psychophysiology of the Lobachevsky State University of Nizhny Novgorod from March to May 2018. The experiment was conducted on twenty-one (21) volunteers aged from nineteen to thirty-six, the average age of the group being twenty-four. Since the largest audience of modern communication networks is the younger generation, they became the focus of the study. The authors analyzed the deformations of the cognitive attitudes of individuals to identify distinctive features of these processes.

Keywords: cognitive attitudes, information influence, emotional maladjustment, telemetry of heart rate, survey.

New Fractal Features for Textural Morphologic Analysis

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Significantly increased resolution of image formation systems (down to a few centimeters) causes a possibility of more effective using of objects textural features and signs in case of thematic processing of radar and optical images [1]. The existing methods of image fractal features measurement allows to evaluate numerically the following topological characteristics of image texture: fractal dimension FD; directional FD in the analysis directions (DFD); multifractal dimension MFD (a widespread case – the spectrum of Renyi dimensions (SRD)); morphological multifractal exponent (MME); fractal signature FS and directional FS (DFS); morphological MFS (MMFS) and lacunarity [1 - 3]. However today there are no complex methods allowing to measure at the same time parameters of the scaling, multifractal and anisotropic properties of a texture possessing reciprocal relationships. In this work the specificities of new Directional Multifractal Blanket method (morphological) (DMBMM) [4] for fractal features measurement of an image textures synthesized on the basis of two best ABRG and MBMM methods in the groups, are considered. Simultaneous accounting of multifractal, singular and

anisotropic properties of the image texture with limited scaling character allowed to increase measuring accuracy both FD, and FS at each analysis scale. This feature is the most representative on comparing with all features considered in this work as the functional correlation of the derived features. The increased informativeness of the developed feature in case of image processing is caused by additional determination, along with multifractal and singular properties, anisotropic properties and their joint account and implied the possibility of its using for the properties description of different images textures and also in images clustering and segmentation tasks.

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Keywords: Fractal dimension, Multifractal signature, Directional features, Anisotropic textures.

Practical Application of New Fractal Features

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The multifractal, singular and anisotropic character of real images can be most fully described by new fractal feature – a directional morphological

multifractal signature (DMMFS) [1]. The research of the DMMFS informativeness measure was conducted by means of comparing of image segmentation efficiency by the FCM (Fuzzy C-Means) clustering algorithm [2], reached when using of the offered textural and fractal feature – DMMFS with reduced dimensionality and MMFS, last of which, according to results of the researches provided in [3] is one of the most informative fractal features of high resolution image texture. Processed grayscale images consisting the images: isotropic fractal Brownian surface (IFBS) with different FD values in the range from 2.1 to 2.9, anisotropic fractal Brownian surfaces model (AFBS), textural 2- and 4-segment mosaics of 50 textures from the Brodatz textures album, anisotropic self-structured surface model (ASSP) and radar images received by means of an air- and space-based X- and Ku-band synthetic aperture radar (SAR) with different resolution. On the basis of the experimental segmentation accuracy assessment of the test, synthesized and real images found that: a) it is not reached an essential difference in results between developed and compared by textural and fractal features when processing the fractal images having the isotropic character; b) increase in segmentation accuracy up to 86,5% is provided when processing anisotropic fractal images; c) improving of segmentation accuracy for 35,6% is reached when processing high-detail images from the Brodatz textures album having multifractal properties, and in that case when the single differentiating characteristic for the segmented images is information about angular dependence of elements of their texture, the accuracy of segmentation increases for 78%; d) using of DMMFS in fuzzy clustering tasks allows to reach more exact clusters division when processing real radar images: in this case it is possible to reach improving of divisibility of different areas up to 63,5%. The greatest gain of segmentation accuracy is watched in case of image processing of the land surface with agricultural purpose sections.

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Keywords: Anisotropic textures, Radar image, Segmentation, Fuzzy C-Means.

A Modified Chaos Game Representation based phylogeny analysis of SARS-COV-2

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We developed a compact and computationally inexpensive method for in-silico comparison of nucleotide sequences at a macro level using subtraction-percentage plots (SP-plots) of a modified chaos game representation (CGR). Analyzing these plots, we defined the k-mer proximity index quantifying the differences between SARS-CoV-2 and other pathogens' genome sequences. We categorized 31 pathogens, based on their proximity to SARS-CoV-2, in four groups to possibly plan a treatment strategy for Covid-19.

Keywords: COVID-19, Chaos Game, Pathogens, Phylogeny.

The role of the angular momentum in shaping collective effects

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It is well-known that classical mechanics is based on the concept of point-like particles. The main laws in physics and mechanics are the laws of conservation of mass, momentum, energy, angular momentum, charge, and some others. In quantum mechanics, the laws of conservation of mass and energy are considered. In our opinion, insufficient attention is paid to the law of conservation of angular momentum. In the report it is shown that the sum of the forces is insufficient for a complete description of the interacting particles. Any redistribution of particles is accompanied by the emergence of collective effects, which is associated with the action of the angular momentum (angular momentum) and, consequently, with the action of an additional force. The effect always manifests itself, regardless of the branch of science: the formation of fluctuations, structures, quantum mechanics and some others. When constructing a theory, it is impossible to restrict oneself to potential forces that depend only on the distance between particles, since when the particles move, the center of inertia shifts, forming a moment. In continuum mechanics, for example, the stress tensor loses its symmetry for this reason.

Keywords: Angular Momentum, Non-Symmetrical Stress Tensor, collective interaction.

Analysis of rogue waves in quantum optics

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In optical non-linear processes, rogue waves can be observed, which can be mathematically described by heavy-tailed distributions. These distributions are special since the probability of registering extremely high intensities is significantly higher than for the exponential distribution, which is most commonly observed in statistical and quantum optics. Our work provides a practical overview of the generic statistics toolkit concerning

heavy-tailed distributions and proposes methods to deal with issues specific to non-linear optics. We take a closer look at supercontinuum generation, where rogue waves were already observed [1]. In our work [2], we have proposed methods to estimate the tail exponent directly in the presence of detector saturation and corrections needed for when the process is pumped by bright squeezed vacuum. In contrast to direct estimation, we have introduced an approach that incorporates more model parameters and estimates them all at once to gain a better understanding of the process. We compare the results of the two approaches, and discuss the consequences in a broader context. The suggested methodology facilitates statistically reliable observation of heavy-tailed distribution in non-linear optics, nano-optics, atomic, solid-state processes, and optomechanics.

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Keywords: Rogue waves, Heavy-tailed distributions, Non-linear optics, Direct estimation vs. modeling.

Tiding up the chaos with Genetic Algorithms: examples in Magnetically Confined Nuclear Fusion

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Magnetically Confined Nuclear Fusion (MCNF) devices produce massive amounts of data, frequently redundant, affected by noise and sometimes corrupted by measurement errors. That data is used to create models in order to detect or to predict specific physical phenomena. Genetic Algorithms (GAs) can be applied to identify and select only the most relevant parameters to be included in these models. By this way, simple equations that summarize the main physics, and therefore the main causal relations, involved in these phenomena can be extracted. In this work, 2

examples of relevant events that occur in the most relevant MCNF device in operation in the world (the Joint European Torus) are addressed: Confinement Regime Classification and Disruption Prediction. With the combination of Support Vector Machines and GAs, linear equations, helpful to provide a simplified landscape of each one of these complex and extremely non-linear phenomena, are reached.

Keywords: Genetic Algorithms, Disruptions, Confinement Regime Identification.

The influence of valence space nucleons on the nuclear structure based on quantum chaos theory

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Multifractal analysis, Localization length (IH) and a number of principal components (NPC) are important measures for chaotic analysis of wave functions in the interacting quantum many-particle systems. Comparing these results with predictions of statistical analysis of energy levels also provides good information about the complexity of these systems. The atomic nuclei is a quantum many-body system with a strong interaction between nucleons. The study of the nuclear structure in the framework of the shell model can amplify features of this interaction. In current work, multifractal, IH and NPC have been computed for specific $J\pi$ to consider the wave functions of nuclear systems. For this purpose, ^{56}Ni and ^{56}Ca with several valence nucleons above $Z=20$ and $N=28$ filled shells of Ca are selected. Analysis of energy levels shows the Wignerian behavior for ^{56}Ni and it is close to the Poisson limit for ^{56}Ca . Results are in good agreement with wave function analysis. The wave functions in ^{56}Ca show more localized behavior compare to ^{56}Ni .

Keywords: Localization length, Statistical analysis, Multifractal, Nuclear shell model, Many-body system.

Neural Networks for Lorenz Map Prediction.

A Trip through Time

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In this article the Lorenz dynamical system is revived and revisited and the current state of the art results for one step ahead forecasting for the Lorenz trajectories are published. Multitask learning is shown to help learning the hard to learn z trajectory. The article is a reflection upon the evolution of neural networks with respect to the prediction performance on this canonical task.

Mutual information rate, Kolmogorov-Sinai entropy and synchronization in cyclic networks with discontinuous local dynamics

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The mathematical information theory studies the quantification, storage and communication of information, highlighting the quantities that are known as information measures, their properties and applications. In addition to its clear importance in the area of telecommunications, information theory has several applications in other scientific and technological areas such as: biology (computational neuroscience), physics (quantum computing), chemistry (intercellular communication) and mathematics (statistical inference, cryptography, network theory and graph theory). The mutual information rate measures the amount of information produced by a network. This rate measures the flow of information between channels connected in a network. Another crucial measure in information theory is the Kolmogorov-Sinai entropy, which

quantifies the uncertainty that defines information. These measures depend on several factors, namely the network topology and the local dynamics, and both are expressed in terms of the conditional Lyapunov exponents. It is also a well known fact that chaotic systems can be synchronized. Motivated by the theoretical and practical connection between the information measures and the synchronization phenomenon, our purpose in this work is to analyze the relations between the mutual information rate, the Kolmogorov-Sinai entropy and the synchronization in the cyclic networks of order N , since information theory and synchronization are directly related in a network. The discontinuous local dynamics considered at each node establishes the topological, metrical and chaotic complexity of the network that is being studied. A similar study has already been performed for complete dynamical networks, see [3], [4] and [5]. Discontinuous dynamical systems are recurrently found in physical systems and are used in several applications in various fields such as engineering, economic, biological and ecological models, among others. The study of discontinuous dynamics in synchronization phenomena has also attracted the attention of several researchers, see, for example, [1] and [2].

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Keywords: Cyclic dynamical networks, discontinuous dynamics, synchronization, information theory, Lyapunov exponents.

On the Potential of Time Delay Neural Networks to identify Causality Graphs

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The detection of causality between physical systems is of great interest in physics and complex sciences, since it may help investigating, modelling, and predicting phenomena still not fully understood. However, finding and measuring the coupling between physical systems remains a difficult task, and a general method, appropriate for every case has not been identified yet. This work presents a new causality detection method based on Time Delay Neural Networks (TDNNs). Using TDNNs to predict one system, on the basis of its past and the past of others, it is possible to detect and quantify, which system influences the predicted one, identifying the exact causality graphs. Some of the most common and critical systems will be analyzed in this work, and the great performances of this new method will be discussed. The proposed approach has also been tested varying the noise of the signals and the number of data to perform the analysis, in order to provide a detailed overview of the limits and potentialities of TDNNs.

Keywords: Causality detection, Chaotic systems, Coupled systems, Time Delay Neural Networks, Time Series.

Non-autonomous two channel chaotic generator: computer modelling, analysis and practical realization

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Circuit of the non-autonomous two channel chaotic generator that contains two operational amplifiers, four capacitors, two resistors and two sinusoidal voltage sources is presented. Regimes of chaotic behavior was modeled by using NI's software MultiSim. Analysis of chaotic attractor, time series and spectra are shown. Designed layout and experimental results of realization of the non-autonomous two channel chaotic generator are presented.

Keywords: Non-Autonomous, Chaotic Generator, Two-Channel, MultiSim.

Competition between Chimeras and Solitary States in Multiplex Neural Networks

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A ring of nonlocally coupled FitzHugh-Nagumo neurons can demonstrate chimera states and solitary states depending on the system parameters. In this work we investigate numerically the dynamics of a two-layer network of the FitzHugh-Nagumo oscillators. Different regimes are chosen for each layer: solitary states in the first ring and chimera states in the second one. We find that the coupled ensembles can be synchronized in both spatiotemporal regimes within different regions of the parameters and with different degrees of synchronization. It is shown that the chimera states can be synchronized independently on initial conditions in comparison with the solitary states. Moreover, we study differences in the network dynamics on the inter-layer coupling via the fast and the slow variables. Our studies show that in the case of the slow variable coupling, the regime

of solitary states (established initially in the first layer) turns out to be dominant in the behavior of the whole network and controls the dynamics of the second layer.

This work was supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation)—Projektnummer 163436311—SFB 910 and by the RFBR and the DFG according to the Research Project No. 20-52-12004.

Keywords: Multilayer network, Synchronization, Chimeras, Solitary states, FitzHugh-Nagumo neurons.

Core expansion and spiral breakup in oscillatory recovering media

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We studied a new type of meandering of spiral waves in an oscillatory recovering media using the modified Barkley's model. The core region expands and the spiral tip follows itself an outward spiral-like trajectory. The spiral waves then break up near the center of rotation, leading to spatiotemporal irregularity. This mechanism, as pointed out by Garfinkel, is still unknown. We investigate the effect of the local dynamics and fixed points, and study the transition to meandering and core expansion as the control parameter is varied. The resulting non-monotonic dispersion curve has implications about similar behavior observed in the restitution curve and attributed to double-wave reentry by other authors.

Socio-Economic Consequences and Health Inequalities in the Light of the Pandemic Covid-19.

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In this report we discuss initially the economic and social determinants of health that have been recorded, such as family environment, early childhood, income, work, unemployment, living conditions, stress, social

support, education level and behavioral factors -smoking, healthy eating and exercise. Further we report how these determinants interact with Covid-19, strengthening economic inequalities with catastrophic consequences worldwide, as expected to have a negative impact on morbidity and mortality rates. Also reference is made how socioeconomic factors affect chronic diseases, with Covid-19 acting as a catalyst, highlighting how unprepared was both the global community and the health systems to deal with an epidemic. As health is the most valuable asset, it is necessary to take measures to reduce inequalities in morbidity or mortality due to Covid-19 that based on policies of prevention-awareness and repression, a number of which are suggested in this article.

Keywords: Socio-economic Factors, COVID-19, Pandemic, Health Inequalities, Chronic Diseases

Implication of Chaos Theory in Startup Entrepreneurship

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Entrepreneurship has many critical factors for success across globe. These factors are more important for “startup” organization. Chaos theory provides a vital perspective on succession rate for “start-up” lifecycle model. This paper creates a framework for analyzing different success parameters e.g. timing, team, idea, business model, funds while highlighting the other vital elements turbulence and unpredictability. Researcher took the data set from few “startups” in the area of EduTech, HealthCare, Automobile and gave an analysis based on chaos theory to depict the competitive advantage over other “startup” with respect to five factors for success.

Keywords: Chaos, Strategy, Startup, Lorenz, Standard deviation.

Chaotic Behavior in a Duopoly Market and application of the d-Backtest Method

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This paper investigates the dynamics of a nonlinear duopoly game with bounded rational players and differentiated goods. We suppose linear demand and the cost functions are derived from the study of a duopoly in the Greek market (two oil companies Hellenic Petroleum and Motor Oil). The game is modeled with a system of two difference equations. Existence and stability of equilibrium of this system are studied. We examine the effect of the parameters in the dynamic of the model and we show that the model gives more complex chaotic and unpredictable trajectories as a consequence of change in the parameter of speed of adjustment and in the parameter of the product differentiation. The chaotic features are justified numerically via computing Lyapunov numbers, sensitive dependence on initial conditions, bifurcations diagrams and strange attractors. Finally, we apply the d-backtest method.

Keywords: Dupoly game; Discrete dynamical system; Nash equilibrium; Stability; Bifurcation; Chaotic Behavior; d-Backtest Method.

On a Cournot Duopoly Game with Relative Profit Maximization

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In this paper we investigate the dynamics of a nonlinear Cournot-type duopoly game with homogeneous goods and heterogeneous expectations. The authors investigate the case, where managers have a variety of attitudes toward relative performance that are indexed by their type. In this game they suppose a linear demand and quadratic cost functions. The game is modeled with a system of two difference equations. Existence and stability of equilibria of the system are studied. The authors show that the models give more complex, chaotic and unpredictable

trajectories, as a consequence of change in the parameter k of speed of the player's adjustment. The chaotic features are justified numerically via computing Lyapunov numbers and sensitive dependence on initial conditions.

Keywords: Cournot duopoly game; Relative profit maximization; Discrete dynamical system; Nash equilibrium; Stability; Bifurcation diagrams; Lyapunov numbers; Strange attractors; Chaotic Behavior.

On the Dynamics of a Bertrand Game with Homogeneous Expectations and Generalized Relative Profit Maximization

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In this paper the dynamics of a nonlinear Bertrand duopoly game with differentiated goods are analyzed. The model describes a competition between two firms whose main interest is to maximize their generalized relative profits. The game is modeled with a system of two difference equations. The parameter's effect on the equilibrium points of the model is examined and their stability conditions are studied. Complex dynamic features including period doubling bifurcations of the unique Nash Equilibrium are also investigated. Numerical simulations are carried out to show the complex behavior. The chaotic features are justified numerically via computing Lyapunov numbers, sensitive dependence on initial conditions, bifurcation diagrams and strange attractors.

Keywords: Bertrand duopoly game, Discrete dynamical system, Homogeneous expectations, Chaotic behavior.

Mixed type of attractor and visualization of system dynamics in reverse time in a physical experiment

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One of the new directions of research in nonlinear dynamics is associated with the study of the so-called mixed dynamics, in which the phase trajectory of a dynamical system visits the regions of phase space corresponding to a chaotic attractor and repeller. The possibility of such behavior has been rigorously proved mathematically and was noted in a number of works, for example, in [1-3]. It seems interesting and important to observe this type of behavior in a real physical system and to visualize the dynamics of the system on a repeller. The aim of this work is to develop a technique for observing repellers in a physical experiment and visualization of mixed dynamics using the example of electronic systems. Observing a repeller in a numerical experiment does not present any particular difficulties, since it is easy to run a program for the numerical integration of differential equations in reverse time or to rewrite equations with negative time and carry out the numerical integration procedure. This is impossible in a physical system. However, having rewritten the equations of a dynamical system, in which the time t is replaced by $-t$, and realizing a similar system, such an object is observed when analyzing the dynamics in reverse time, since in this case the unstable manifolds become stable. In [4], a simple model was proposed in the form of a system of three nonlinear differential equations, the solutions of which are determined on a compact set. To achieve this goal, the equations were written in reverse time, and electronic circuits were implemented for each of the systems. As a result, the first diagram showed the dynamics of the model in forward time, and the second diagram showed the dynamics that is equivalent to the dynamics of the first diagram in reverse time. In this case, the repeller, which takes place in the dynamics of the first scheme, is an attractor in the dynamics of the second scheme. In this case, the attractor observed in the dynamics of the first scheme will be a repeller, which takes place in the dynamics of the second scheme. As a result, using the example of a simple three-dimensional system, a technique was proposed for observing repulsive sets in a physical experiment, visualization of a repeller was carried out, and for the first time in a physical experiment, the results of an experimental study of a new

type of chaotic behavior representing motion on an attractor and repeller were presented. The results of the physical experiment are in good agreement with the results of numerical studies.

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Forced van der Pol oscillator - synchronization from the bifurcation theory point of view

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The contribution presents a bifurcation theory point of view to synchronization of a forced van der Pol oscillator, which is coupled to a master oscillator as a system with a stable limit cycle corresponding to harmonic oscillation. We present bifurcation manifolds, 3D sections of the phase space and its Poincaré sections for parameters close to these manifolds providing a clear visualization of the dynamics of the 4D system. Among other things, we present the coexistence of a stable torus and a stable cycle arising from m-fold bifurcation on an Arnold's tongue.

Two Types of Sensitivities within Lorenz Models

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Following Lorenz's studies (Lorenz 1963, 1969, 1972), deterministic, chaotic solutions and closurebased, linearly unstable solutions have been an explicit and/or implicit focus for understanding predictability in weather and climate. In this talk, I will discuss various types of solutions, two kinds of attractor coexistence, and two types of sensitivities within Lorenz models (Lorenz 1963 and 1969; Shen 2019) in support of the revised view on the dual nature of chaos and order in weather and climate (Shen et al. 2021a, b).

Anti-phase synchronization in networks of repulsively coupled 2D lattices of oscillators

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We for the first time explore numerically the effect of anti-phase synchronization in a heterogeneous two-layer network of two-dimensional (2D) lattices of van der Pol oscillators. The inter-layer coupling of the multiplex network has linear repulsive character. One layer of 2D lattices is characterized by attractive coupling of oscillators and demonstrates a spiral wave regime. The oscillators in the second layer are coupled through active elements and the interaction between them has the repulsive character. We show that the lattice with the repulsive type of coupling demonstrates complex spatiotemporal cluster structures, which can be called as labyrinth-like structures. We show for the first time that this multiplex network with fundamentally different types of intra-layer coupling demonstrates the inter-layer anti-phase synchronization and a competition between two types of structures. Our numerical study indicates that the synchronization threshold and the type of spatiotemporal patterns in both the layers strongly depend on the ratio of the intra-layer coupling strength of the two lattices.

Keywords: Synchronization, multiplex networks, spatiotemporal patterns, repulsive coupling, van der Pol oscillator, spiral wave.

Fractal Nanoparticles of Phase-Separating Solid Solutions: Nanoscale Effects on Phase Equilibria, Thermal Conductivity, Thermoelectric Performance

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In recent years, thermoelectric materials and energy converters have attracted considerable interest as an essential part of the “green energy” technologies [1]. One of the most promising ways of obtaining high values of thermoelectric figure of merit is the formation of nanostructured 3D materials with nanoparticles of phase-separating alloys. Using the example of low-temperature thermoelectric Bix-Sb1-x alloys for the application in space engineering [2], we have shown how nanoscale effects on phase equilibria in nanoparticles influence on their thermoelectric properties. Such effects consist in nonlinear changes in mutual solubilities of components at a given temperature, phase transition temperatures and even the total suppression of the phase separation depending on the size and shape of a nanoparticle as well as its surrounding environment. The combination of thermodynamic and ab initio approaches has been used while the nanoparticle shape has been determined using the methods of fractal geometry [3,4]. The considered phenomena have been found to be driven by several mechanisms, the interpretation of which has been presented. For each operating temperature, we have found the optimal range of sizes and shapes (fractal dimensions) of nanoparticles, at which their equilibrium phase composition leads to a dramatic reduction of the phonon thermal conductivity and a notable increase in their thermoelectric figure of merit. In the report, we have also suggested a method of calculating the equilibrium size and shape distributions of nanoparticles at given thermodynamic conditions by a combined usage of number theory, fractal geometry and statistical thermodynamics and verified its accuracy by analyzing a large set of latest experimental data. In the last section, we

have presented a brief review of the latest results concerning the problem of grain boundary phonon scattering which has an additional influence on the thermoelectric performance of 3D bulk materials formed with nanoparticles by using the up-to-date additive technologies.

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Keywords: Nanoparticles, Fractals, Nanothermodynamics, Thermoelectricity, Phonon Thermal Conductivity, Size Distributions.

Maximal attractors in nonideal hydrodynamic systems

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Some nonideal hydrodynamic systems of the type "tank with liquid - source of excitation of oscillation" are considered. New types of limit sets of such systems, so called maximal attractors, have been discovered and described. It was found that the maximal attractors can be both regular and chaotic. Main characteristics of the described maximal attractors are analyzed in details. Transitions to deterministic chaos in such systems are considered. Despite the fact that maximal attractors are not attractors in the traditional sense of this term, it is shown that the transition from regular maximal attractors to chaotic maximal attractors can occur for "usual" attractors.

Keywords: nonideal hydrodynamic systems, maximal attractors, scenarios of transition to chaos.

Various oscillatory modes of spontaneous calcium concentration in astrocytes

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The brain is a complex multicomponent system, the main elements of which are neurons and glial cells. It is known that the processes of generation, transmission and transformation of electrical impulses and their sequences in neural systems are the basis for information processing in the brain. At the same time, a lot of data obtained experimentally demonstrate that the role of glial cells in these processes is of particular significance [1-3]. In this work, the bifurcation mechanisms for oscillatory mode emergence in dynamics of astrocytic spontaneous calcium concentration have been studied. With this aim a mathematical model of Lavrentovich and Hemkin calcium signaling pathways has been considered [4]. Two types of oscillatory regime emergence have been observed: change of the parameters can lead either to birth of a small-amplitude spontaneous oscillations or sudden large-amplitude chemical activity. Bistable type of the system behavior has been revealed. The impact of extracellular calcium concentration on characteristics of such behavior has been studied. Emergence of chaotic regime in dynamics of astrocytic spontaneous calcium concentration has also been examined. Complicated modes of oscillatory calcium dynamics are of particular importance because such type of astrocytic chemical signals can significantly affect the dynamics of neurons and modify the neural network response.

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The work was supported by the Ministry of Science and Higher Education of the Russian Federation (Project No. 14.Y26.31.0022)

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Keywords: Mathematical modeling, Calcium concentration in astrocytes, Oscillatory and stationary modes, Bistability mode, Calcium oscillator, Bifurcation, Astrocyte.

Invariant Model of Boltzmann Statistical Mechanics, Generalized Thermodynamics, and Shannon Information Theory

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Further implications of an invariant model of Boltzmann statistical mechanics to generalized thermodynamics, quantum mechanics, chaos, and Shannon information theory are studied. The nature of matter and Dirac anti-matter are described in terms of states of compression and rarefaction of physical space, Aristotle fifth element, or Casimir vacuum identified as a compressible tachyonic fluid. The model is in harmony with perceptions of Plato who believed that the world was formed from a formless primordial void that was initially in a state of total chaos or "Tohu Vavohu" [Int. J. Mech. 8, 73-84]. Hierarchies of statistical fields from photonic to cosmic scales lead to universal scale-invariant Schrödinger equation thus allowing for new perspectives regarding connections between classical mechanics, quantum mechanics, and chaos theory. The nature of external physical time and its connections to internal thermodynamics time, and Rovelli thermal time as well as time irreversibility are described. Finally, some implications of renormalized Maxwell-Boltzmann distribution function to social and economic systems will be discussed.

Keywords: Thermodynamics, Quantum mechanics, Anti-matter, Spacetime, Thermal time, Time irreversibility.

D-Entropy in classical mechanics

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The physical essence of D-entropy, obtained directly from the dynamic characteristics of the system, determined by the fundamental laws of classical mechanics, is discussed. A distinctive feature of D-entropy is that it determines the change in entropy of a body that occurs as a result of its motion in an external non-uniform field of forces. The concept of D-entropy in classical mechanics follows from the equation of motion of structured bodies. This equation of motion is derived based on the principle of symmetry dualism. According to this principle, the evolution of structured bodies is determined by both the symmetry of space and the symmetry of the body. Therefore in accordance with the principle of dualism of symmetry, the energy of the body should be represented as the sum of the internal energy of the body and the energy of its movement. Such a representation of energy is carried out in the space of micro-variables that determine the movement of the system's elements, and macro-variables that determine the movement of the system, respectively. The use of the principle of symmetry dualism made it possible to take into account in the equation of motion nonlinear terms that determine the transformation of the energy of motion of a body into the internal energy of motion of its elements. This transformation is determined by bilinear terms depending on micro- and macro-variables. These terms arise when the body moves in an inhomogeneous space. The physical meaning of D-entropy is that it determines the efficiency of this of this energy conversion. D-entropy is applicable for bodies of large and small number of elements. The D-entropy for large equilibrium systems, like the Clausius entropy, only increases. For small systems, the D-entropy may decrease. Since the D-entropy is obtained from general equations of dynamics, it can be used to determine the areas of applicability of statistical laws, as well as the applicability of entropy obtained by statistical means. The main advantage of D-entropy is that it is determined by the nature of the external impact on the system and its internal state during the movement of the system in an inhomogeneous field of external forces. It was shown how D-entropy can be used to study the processes of evolution of matter which is submitted as a hierarchy of open nonequilibrium dynamical systems.

Keywords: entropy, symmetry, irreversibility, evolution, mechanics.

A Stochastic Compartmental Model for COVID-19

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We propose two stochastic models for the Coronavirus pandemic. The statistical properties of the models, in particular the correlation functions and the probability density function, have duly been computed. Our models, which generalises a model previously proposed and published in a specialised journal, take into account the adoption of the lockdown measures as well as the crucial role of the hospitals and Health Care Institutes. To accomplish this work, we have analysed two scenarios: the SIS-model (Susceptible \Rightarrow Infectious \Rightarrow Susceptible) in presence of the lockdown measures and the SIS-model integrated with the action of the hospitals (always in presence of the lockdown measures). We show that in the case of the pure SIS-model, once the lockdown measures are removed, the Coronavirus will start growing again. However, in the second scenario, beyond a certain threshold of the hospital capacities, the Coronavirus is not only kept under control, but its capacity to spread tends to diminish in time. Therefore, the combined effect of the lockdown measures with the action of the hospitals and health Institutes is able to contain and dampen the spread of the SARS-CoV-2 epidemic. This result can be used during a period of time when the massive distribution of delivery of a limited number of vaccines in a given population is not yet feasible. By way of example, we analysed the data for USA and France where the intensities of the noise have been estimated by Statistical Mechanics. In particular, for USA we have analysed two possible hypotheses: USA is still subject to the first wave of infection by Coronavirus, and USA is in the second (or third) wave of SARS-CoV-2 infection. The agreement between theoretical predictions and real data confirms the validity of our approach.

Keywords: Mathematical model; COVID-19; Dynamics of population; Pneumonia.

Note: The full manuscript may be found at the address: <http://arxiv.org/abs/2012.01869>.

Scenario of occurrence of chaos with additional zero Lyapunov exponent in flow systems

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Chaotic dynamics can be verified and classified with the spectrum of Lyapunov exponents. In the case when the number of positive Lyapunov exponents more than one, oscillatory mode is called hyperchaos. It is also possible to implement such chaotic behavior when spectrum of Lyapunov exponents contain one positive and two zero Lyapunov exponents. In the frame of this work universal scenario of formation of chaotic attractors with additional zero Lyapunov exponent is discussed, which involve appearance of a set of saddle tori via cascade of tori doubling bifurcations and absorption of these saddle tori by chaotic attractor. The scenario is demonstrated on the example models of different nature.

The work is supported by the grant of the RFBR (Project No. 19-31-60030).

Keywords: chaos, Lyapunov exponents, flow dynamical system.

Global dynamics of a business-cycle modeled by a triangular map

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The mathematical model of the neoclassical one-sector growth dynamics with differential savings and non-constant labor force growth has been

proposed as a bidimensional triangular map which depends on several parameters being one of them, the elasticity of substitution, a key for the appearance of complex dynamics. In this work, the labor force growth evolution is described by a generalized logistic, a discrete time version of the Richard's equation, which involves an extra parameter for better adjustment of the model and so, extending the proposal of [1]. Our aim is to study the global dynamics of the resulting map for all the plausible values of the parameters. Concepts like absorbing set and global attractor as stated in [2] will be useful when applying the methodology taken from [3]. Known facts on the existence and stability of fixed points and periodic orbits of one-dimensional maps have been shown to be also valid for bi-dimensional triangular maps in ([4]) so they could be applied on this case. By the last, the bi-dimensional triangular nature of the map would help to provide more evidence on complex dynamics as cycles of every order or chaotic behavior.

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Keywords: neoclassical one-sector growth, triangular map, generalized logistic, fixed points, periodic orbits, absorbing set, global attractor, chaotic behaviour.

Synchronization of Chaotic Colpitts Oscillators

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Nowadays, many telecommunication systems are based on chaos. Chaotic systems have a number of characteristics suitable for telecommunications. One of these characteristics is the ability to synchronization. The focus, however, is on the synchronization between two chaotic Colpitts oscillators. This paper explores the development of a system synchronizing an ideal and a pragmatic oscillator, achieving Colpitts oscillator. The system displays a unique characteristic in reference to the synchronization of the two oscillators, as they have been implemented differently. The first oscillator has been developed via software, while the second oscillator has been developed via hardware.
Keywords: Synchronization of Colpitts Oscillators, Chaotic Behavior of Colpitts Oscillator, Colpitts Attractor, Chaos.

Positive Solutions for a Singular Fractional Boundary Value Problem

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We investigate the existence of positive solutions for a system of Riemann-Liouville fractional differential equations, supplemented with uncoupled nonlocal boundary conditions which contain various fractional derivatives and Riemann-Stieltjes integrals, and the nonlinearities of the system are nonnegative functions and they may be singular at the time variable. In the proof of our main theorems we use the Guo-Krasnosel'skii fixed point theorem.

Keywords: Systems of Riemann-Liouville fractional differential equations, Nonlocal boundary conditions, Singular nonlinearities, Existence of solutions, multiplicity.

Some aspects of Rainbows and Black Holes linked to Mandelbrot set and Farey diagram

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We are presenting some parallels between geometric optics and Mandelbrot set based on the existence of star polygons trajectories in billiards. In this work, we explore the dynamics of a light propagating in a cylinder inside a cylinder using Farey mediant [1]. This dynamics is strongly correlated with the trajectories of star polygons in a billiard. The star polygons are also observed in the complex plane related to Mandelbrot set, obtained from an iteration rule [2]. In this scenario, the trajectories in the complex plane are divided in regions or domains which exhibit different solutions such as periodic, quasi-periodic and chaotic, related to the Mandelbrot set, resembling properties of the dynamics of two-frequency torus breakdown. We compare these solutions with optical caustics observed in rainbows and image formation of gravitational lenses based on black holes.

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Forecasting the equity risk premium with long swings in stock market behaviour

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This paper shows that substantial change in stock market behaviour has a statistically and economically significant impact on equity risk premium predictability in-sample and out-of-sample. The change in stock market behaviour is measured by the local Hurst exponent using multifractal detrending moving average analysis (MFDMA) and stock market returns. Our findings suggest that after a positive shock, the stock market tends to approach its fundamental value in line with the macroeconomic variables for up to six months. In contrast, the stock market tends to be valued with respect to its past price performance measured by the technical indicators after a negative shock for up to nine months. In asset allocation analysis, we show that investors can exploit this information to yield a substantial 12% annualized certainty equivalent return within six months after a shock with forecasts based on both macroeconomic variables and technical

indicators, even when a proportional transactions cost of 50 basis points per transaction is imposed.

Keywords: Hurst exponent, Multifractal Detrending Moving Average Analysis, Equity Risk Premium Predictability, Stock Market Behaviour.

Statistical processes of transformation and accumulation of energy under irradiation of solids

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The paper presents some results of the research of such processes, based on our own long-term thematic experimental and theoretical experience. They are presented as the results of the interaction of statistical ensembles on one side of the radiation source, and on the other side of the radiation environment [1,2]. The main problem is in the following fact that the characteristics of these ensembles change significantly upon irradiation. Especially when we have deal with intense pulsed radiation, such as laser beams and charged particles of electrons and ions. Serious difficulties arise when we calculate of the absorbed radiation dose for example, by Monte Carlo methods. And in application of theory of branching processes in the calculation of phase transition processes of the first and second order in mechanical fracture of solids.

The reliability of the studies cited is confirmed by the correspondence of theoretical calculations based on physical and mathematical models to experimental facts.

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Pseudorandom numbers generating in the unit circle $|z| < 1$

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Let $p \equiv 3p \pmod{4}$ be the prime rational number, 1 be the natural number. Let consider over the ring of Gaussian integers $Z[i]$ the elliptic curve given by the congruence

$$y^2 \equiv x^3 + ax + p^m, \quad (1)$$

where a, b are the Gaussian integers under condition $(4a^3 + 27b^2, p) = 1$. The residue classes modulo p form the field of p^2 elements in $Z[i]$. Let (x_0, y_0) be the solution of congruence (1). We construct the sequence of points $\left(\frac{x_0(n)}{p^n}, \frac{y_0(n)}{p^n}\right)$, $n = 0, 1, 2, \dots$ that passes a serial test on pseudorandomness in the unit circle of complex numbers.

Keywords: pseudorandom numbers, Gaussian integers, elliptic curve, unit circle

Sequences of pseudorandom numbers from elliptic curves on \mathbb{Z}_p^m

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Let $p > t$ be a prime number. We consider the set of solutions of the following congruence

$$y^2 \equiv x^3 + ax + b \pmod{p^m}, \quad (m \in \mathbb{N})$$

For $m = 1$ this set together with the infinity point \mathcal{O} forms the abelian group E_p with elements (x_0, y_0) . For every $(x_0, y_0) \neq \mathcal{O}$ and $(y_0, p) = 1$ we deduce the representation of arbitrary solution of the above congruence in the following form

$$x(t) \equiv x_0 + pt, \quad y(t) \equiv y_i(0)\varphi(t) \pmod{p^m},$$

where $\varphi(t) = \varphi_0(x_0) + p^{\lambda_1}\varphi_1(x_0)t + \dots + p^{\lambda_s}\varphi_s(x_0)t^s$ and $y_i(t), i = 1,2$ is the solutions of the congruence

$$y^2 \equiv x_0^3 + ax_0 + b \pmod{p^m}.$$

For $j \geq 3$ the coefficients $\varphi_j(x_0)$ are determined by the congruential recursion of type

$$\begin{aligned} \varphi_{j+1} \equiv & \frac{2j+1}{2(j+1)}(3x_0^2x'_0 + ax_0)\varphi_j - \frac{3(j-2)}{j+1}x_0x'_0\varphi_{j-1} \\ & - \frac{2j-7}{2(j+1)}x'_0\varphi_{j-2} \pmod{p^m} \end{aligned}$$

with initial values modulo p^m

$$\begin{aligned} \varphi_0 = 1, \quad \varphi_1 = \frac{1}{2}(3x_0^2x'_0 + ax_0), \quad \varphi_2 = \frac{1}{2}3x_0x'_0 - \frac{1}{8}(3x_0^2x'_0 + ax'_0)^2 \\ x'_0(x_0^3 + ax_0 + b) \not\equiv 1 \pmod{p^m} \text{ and also } \lambda_1 = 1, \lambda_2 = 2, \lambda_j \geq 3, \\ j = 3,4, \dots \end{aligned}$$

We prove that for $t = 1,2, \dots, p^m - 1$ the sequence $\left\{\frac{y(t)}{p^m}\right\}$ be the sequence of pseudorandom numbers and passes the test on pseudorandomness and unpredictability. Sequence $\left\{\frac{y_n(t)}{p^m}\right\}, n = 0,1, \dots$ we call the sequence of pseudorandom numbers from elliptic curve on the finite ring \mathbb{Z}_{p^m}

Keywords: pseudorandom numbers, elliptic curves, finite ring, abelian group.

Anomaly Detection and Unsupervised Classification of Plasma Events

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In nuclear fusion experiments, massive databases of time series signals store all relevant information about the plasma evolution. However, most of the physics knowledge remains hidden due to two factors. On the one hand, most measurements are taken by indirect methods and the inversion techniques are quite complex. On the other hand, the plasma shows highly non-linear interactions that are difficult to model. The ITER database is expected to store more than 1 Tbyte of data per discharge (about 1 million signals, mainly time series and video-movies) and the extraction of hidden knowledge will be essential. One of the most important aspects in ITER will be the automatic recognition of off-normal

events as the plasma evolves. A crucial objective is the real-time determination of this kind of events. However, this is an extremely difficult task not tackled so far that requires off-line analysis of the databases to develop proper methods to be applied under real-time requirements. This work presents a three step off-line method to perform the temporal location of anomalies, the unsupervised grouping of events and the potential analysis of causal relationships. An example of the latter can be the sequence of events that can produce different types of plasma disruptions. The first step of the method is the recognition and temporal location of anomalies by analyzing multi-dimensional parameter spaces made up of plasma quantities. The second step consists of determining characteristic time lengths of such anomalies. The third step is the unsupervised classification of the events in order to assign labels to each class of physics event. The unsupervised classification into different classes is used to filter out those groups without statistical relevance.

Keywords: Nuclear fusion, Data mining, Unsupervised classification.

Generalized Lorenz Chaotic System in Optimal Control Model

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Following Benhabib et al. 2003, in this paper, we consider an optimal control model that represents a theoretical evaluation of monetary policy rules that favor backward-looking behavior on the part of the central bank. We analyzed the three-dimensional continuous-time dynamical system generated by the optimization problem. From an analysis of the stability of the multiple equilibria and their eigenvalues, we are able to prove that the autonomous system belongs to a generalized Lorenz class. It might be possible to characterize the system for a full set of parameter spaces; we show the possibility of global indeterminacy and chaos in its subset. A global numerical analysis corroborates our results.

Keywords: Optimal control model, Lorenz class, Chaos Attractor.

Fractals and Chaos in the Fixed Point Trajectories of Nonlinear Dynamical Systems associated with the Wright-Fisher Model and its Generalizations

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It is proved that the Wright-Fisher model of population dynamics can be developed and generalized to the level of dynamical systems models of stock and foreign exchange markets, analysis and processing of big data, and construction of dynamic database management systems. Methods for the analysis of trajectories of cyclic fixed points of generalized Wright-Fisher models are developed. It is proved that the structure of trajectories is adequately described in the language of the fractal and chaos theory in accordance with the laws that are determined by the parameters of the models.

Keywords: Wright-Fisher model, Dynamical systems, Fractals.

Noether Currents for Eulerian Variational Principles in Non-Barotropic Magnetohydrodynamics and Restriction of Chaos by Topological Conservation Laws

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It was shown previously that ideal non-barotropic MHD is mathematically equivalent to a five-function field theory with an induced geometrical structure in the case that field lines cover surfaces, and this theory can be described using a variational principle. Here we use various symmetries of the flow to derive topological constants of motion through using Noether currents associated with label translations and discuss their implication for the stability of non-barotropic MHD configurations and restriction of chaos.

Fractional Chaotic system solutions and their impact on chaotic behavior

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This paper is devoted to the analysis of calculation methods for solving fractional chaotic systems and the impact of these different approaches on the chaotic behavior of the fractional chaotic system. The complex dynamic behavior in some nonlinear systems has been largely discussed due to its intriguing characteristics, being dependent only on systems' initial conditions and parameters. However, most discussions in this domain are centered on the classical integer-order chaotic system. The fractional-order chaotic system, on the other hand, due to its intrinsic complexity in both the analytical point of view and engineering implementations, is relatively less discussed. The difficulties lie in the fact that there are different definitions and approximations for fractional calculus. What's more, the system's outputs and its chaotic behavior are, to some extent, dependent on the chosen solving approach for the fractional differential equations. In this paper, we have oriented our research direction to the different approximations of fractional calculus and different fractional differential equations solving methods. We have first introduced two widely used time domain fractional differential equations solving approaches, the corrector-predictor method based on Caputo fractional derivative definition and the short memory calculation approach based on Grunwald fractional derivative. Then, these numerical solutions calculation methods are employed to depict the phase portrait of a class of commensurate and incommensurate fractional chaotic systems which are extended cases from the classical chaotic ones. The Lyapunov exponent is used as an indicator of systems' chaoticity to analyze the impact of different approaches on the non-linear dynamics of the fractional chaotic systems. The bifurcation diagrams of the systems over various fractional orders and initial conditions are also illustrated to detect the sensitivity changes of the chaotic system applying different calculation approaches. Based on these discussions, we proposed a slightly modified calculation process that is suitable for both commensurate and incommensurate fractional chaotic systems.

Keywords: fractional calculus, numerical solution, fractional chaotic system, non-linear dynamics.

Chaos in Water Resources

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Hydrological systems produce runoff, infiltration, evapotranspiration, and surface storage from precipitation. Through their dependence on the atmosphere for precipitation, hydrologic systems are likely to be chaotic, but it is difficult to say, because the Earth's near-surface and biosphere strongly mediate the precipitation signal. Scientists have sought to quantify whether hydrological systems are in fact chaotic, just as they have for many other kinds of dynamical systems in nature and civilization. Attempts to quantify the chaotic nature of hydrologic systems have largely relied on calculations of the Correlation Dimension (D_c) for given time series using a procedure called Correlation Integral Analysis (CIA). Ideally this method is coupled with other indicators, but this has been the most heavily used metric. If D_c is non-integer then the dimension is fractal and the time series is likely to be chaotic.

Universality in GDP scaling

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In Hastings and Young-Taft (2020) we considered empirically observed GDP scaling over time, finding stable regimes including a high-end power law tail, a middle scaling region where GDP decreases exponentially with rank, and a more rapidly decaying low-end tail, exponentially with rank squared, over the 40 year period from 1980 to date. Montroll and Shlesinger (1982) argued for a basic lognormal distribution as a consequence of multiplying many independent random variables, together with a power law high-end tail (as we later observed) because “the very wealthy generally achieve their superwealth through amplification processes that are not available to most.” We identify universal patterns in GDP scaling by applying curve registration techniques to the evolution of GDP as a function of rank order and time.

References:

Hastings, H.M. and Young-Taft, T. 2020. Empirical scaling and dynamical regimes for GDP: challenges and opportunities. International Conference on Complex Systems, virtual, July 2020.

Montroll, E.W. and Shlesinger, M. F. 1982. On $1/f$ noise and other distributions with long tails. Proceedings of the National Academy of Sciences, 79, 3380-3383.

Keywords: Economic models, Scaling regions, Universality, Curve registration.

Chaos in the Economic System in the Conditions of Innovation and Market Transformations

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The prospects of development of world economy it is accepted to bind to becoming of innovative economy. Innovation as an original form of chaos can become a shove and mechanism of output on one of possible trajectories of development, which corresponds the internal tendency of the economic system and provides its new qualitative state. This is the essential and constructive role of innovators to start the processes of self-organization in the system and preparing it for different scenarios of development. Innovation as a kind of chaos is a factor, which brings the nonlinear system to own structures-attractors. At the same time, the economic system is feeling the influence of fluctuations, which arise up from negative entropy connections from the side of other macrosystem of external environment. A non-stationary external environment, conditioned market changeability, foresees the presence of conjuncture vibrations, which carry chaotic character. Thus, the conduct of the economic system, its structure and management, is formed on crossing a few determined chaos, on the border of which there are processes of self-organization. For research of these processes, the model of origin of the mixing scenario is built in the conditions of innovative changes and market vibrations. The scenario of evolutionary process of transition of the economic system on the verge of chaos is got. The practical criteria of evaluation of finding of the system are built in area of interaction internal and external chaos.

Keywords: Chaotic modeling, The innovative and market changeability problem, Mixing trajectories, Verge of chaos, Simulation, Chaotic simulation.

Coupled Fitzhugh-Nagumo Type Neurons Driven by External Voltage Stimulation

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We have extended some results of previous works on coupled Fitzhugh-Nagumo type neurons stimulated by external alternate voltage source. At first, an electronic circuit modelling a particular system of neurons is constructed. Analysis of dependence between the frequency of the stimulating signal and an emergence of chaos enable us to interpret the stimulating signal as brain alpha waves. For appropriate values of parameters a chimera-like state can be observed.

Keywords: Fitzhugh-Nagumo, neuronal cells, alpha waves, chaos, chimera state.

Special Session: Title and Chair: Synchronizations, Chimeras.

Optimal Control Problem for a Group of Objects

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A hybrid dynamic system consisting of a variable number of control objects is considered. In the initial and final states, the system contains a group of n objects. But in the process of control, the number of objects can change at arbitrary times in accordance with predetermined rules. As a criterion for constructing optimal trajectories for the movement using a minimum of the total energy consumption. This formulation of the problem is similar to the Traveling Salesman Problem in the case when several traveling salesmen walk around the points, and the number of traveling salesmen is not constant. The application of the problem under consideration can be the problem of optimizing the delivery routes of goods to consumers.

Keywords: Optimal Control, Energy Consumption, Operations Research, Traveling Salesman Problem.

On the Energy Harvesting from a Base-Rotating Double Pendulum

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Modern electronic devices and wireless sensor nodes require less energy on-board. Such devices could be powered by energy extracted from the environment using modern energy harvesting techniques. One such harvester that exploits this technique is the excited double pendulum. However, in the presence of mechanical and electrical damping the large amplitude responses can deteriorate over time into low energy chaotic oscillations, thereby reducing the harvested power. Under such conditions, forcing the system to follow a high energy orbit would improve the energy harvested from the system. On this regard, this study focuses on exploiting the dynamics of a novel base-rotated double pendulum. The analytical formulation for the system is established based on the Lagrangian formulation. And, the effect of frequency and length ratio are analyzed under two different configurations; with and without damping.

Keywords: Energy harvesting, Double pendulum, Chaotic modeling.